Differentiation

$y = f(x)$	$\dfrac{dy}{dx} = f'(x)$
k, constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n, any constant n	nx^{n-1}
e^x	$e^x = y$
e^{kx}	$ke^{kx} = ky$
$e^{f(x)}$	$f'(x)e^{f(x)}$
$\ln x$	$1/x$
$\ln kx = \log_e kx$	$1/x$
$\ln f(x)$	$f'(x)/f(x)$

The sum–difference rule

$$\frac{d}{dx}(u(x) \pm v(x)) = \frac{du}{dx} \pm \frac{dv}{dx}$$

Constant multiples

$$\frac{d}{dx}(k \times f(x)) = k \times \frac{df}{dx}$$

for k constant

The product rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

The quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

The chain rule

If $y = y(u)$, where $u = u(x)$, then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

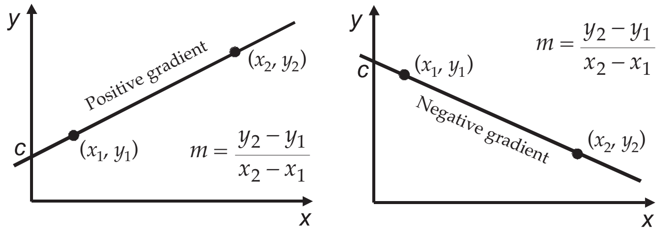
Integration

$f(x)$	$\int f(x)\,dx$
k, (any) constant c	$kx + c$
x	$\dfrac{x^2}{2} + c$
x^2	$\dfrac{x^3}{3} + c$
x^n, $(n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + c$
$x^{-1} = 1/x$	$\ln x + c$
e^x	$e^x + c$
e^{kx}	$\dfrac{e^{kx}}{k} + c$

Graphs of Common Functions

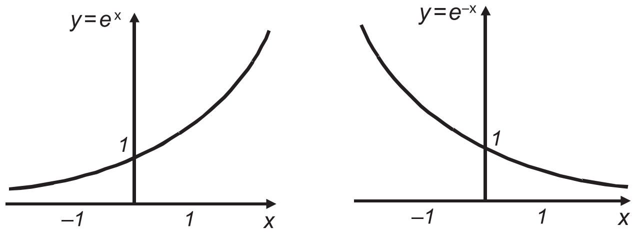
Linear

$y = mx + c$; m = gradient; c = vertical intercept



Exponential functions

$e \approx 2.7183$ is the exponential constant

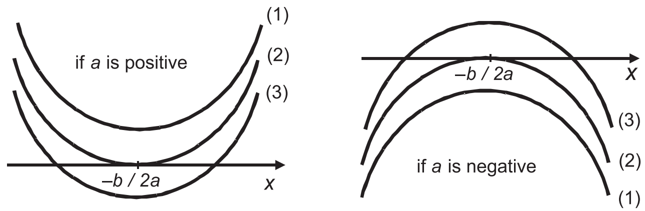


Graph of $y = e^x$ showing exponential growth

Graph of $y = e^{-x}$ showing exponential decay

Quadratic functions

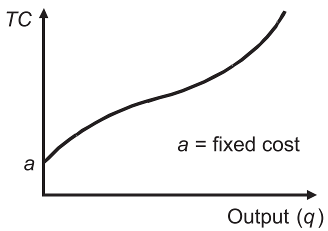
$y = ax^2 + bx + c$



(1) $b^2 - 4ac < 0$; (2) $b^2 - 4ac = 0$; (3) $b^2 - 4ac > 0$

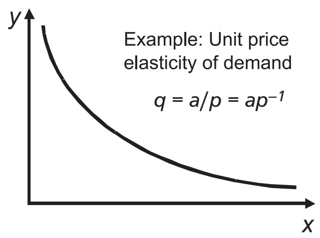
Total cost functions

$TC = a + bq - cq^2 + dq^3$

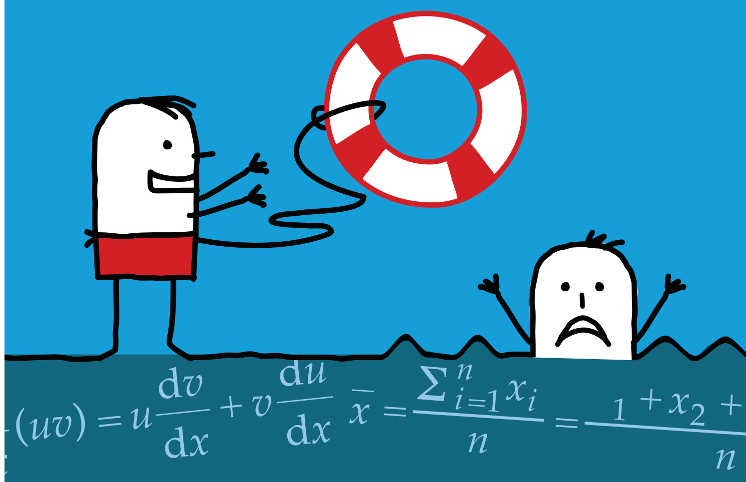


Inverse functions

$y = a/x = ax^{-1}$



Studying Economics



Maths for Economics

PRINCIPLES AND FORMULAE

A lifeline for economics students

www.studyingeconomics.ac.uk

The Higher Education Academy Economics Network

www.economicsnetwork.ac.uk

mathcentre

www.mathcentre.ac.uk

Arithmetic

When multiplying or dividing positive and negative numbers, the sign of the result is given by:

+ and + gives +	e.g. 6 x 3 = 18;	21 ÷ 7 = 3
– and + gives –	e.g. (–6) x 3 = –18	(–21) ÷ 7 = –3
+ and – gives –	e.g. 6 x (–3) = –18	21 ÷ (–7) = –3
– and – gives +	e.g. (–6) x (–3) = 18	(–21) ÷ (–7) = 3

Order of calculation

First: brackets
Second: x and ÷
Third: + and –

Fractions

$$\text{Fraction} = \frac{\text{numerator}}{\text{denominator}}$$

Adding and subtracting fractions

To add or subtract two fractions, first rewrite each fraction so that they have the same denominator. Then, the numerators are added or subtracted as appropriate and the result is divided by the common denominator: e.g.

$$\frac{4}{5}+\frac{3}{4}=\frac{16}{20}+\frac{15}{20}=\frac{31}{20}$$

Multiplying fractions

To multiply two fractions, multiply their numerators and then multiply their denominators: e.g.

$$\frac{3}{7}\times\frac{5}{11}=\frac{15}{77}$$

Dividing fractions

To divide two fractions, invert the second and then multiply: e.g.

$$\frac{3}{5}\div\frac{2}{3}=\frac{3}{5}\times\frac{3}{2}=\frac{9}{10}$$

Series (e.g. for discounting)

$1 + x + x^2 + x^3 + x^4 + \ldots = 1/(1-x)$

$1 + x + x^2 + x^3 + \ldots + x^k = (1-x^{k+1})/(1-x)$

(where $0 < x < 1$)

Algebra

Removing brackets

$a(b + c) = ab + ac$ $\qquad a(b - c) = ab - ac$

$(a + b)(c + d) = ac + ad + bc + bd$

$(a + b)^2 = a^2 + b^2 + 2ab;$ $\qquad (a - b)^2 = a^2 + b^2 - 2ab$

$(a + b)(a - b) = a^2 - b^2$

Formula for solving a quadratic equation

$$\text{If } ax^2 + bx + c = 0, \text{ then } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Laws of indices

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n}=a^{m-n} \qquad (a^m)^n = a^{mn}$$

$$a^0 = 1 \qquad a^{-m}=\frac{1}{a^m} \qquad a^{1/n}=\sqrt[n]{a} \qquad a^{m/n}=\sqrt[n]{a^m}$$

Laws of logarithms

$y = \log_b x$ means $b^y = x$ and b is called the **base**

e.g. $\log_{10} 2 = 0.3010$ means $10^{0.3010} = 2.000$ to 4 sig figures

Logarithms to base e, denoted $\log_e$, or alternatively ln, are called *natural logarithms*. The letter e stands for the exponential constant, which is approximately 2.7183.

$$\ln AB = \ln A + \ln B \;;\quad \ln\frac{A}{B} = \ln A - \ln B \;;\quad \ln A^n = n\ln A$$

Proportion and Percentage

To convert a fraction into a percentage, multiply by 100 and express the result as a percentage. An example is:

$$\frac{5}{8} \text{ as a percentage is } \frac{5}{8}\times 100 = 62.5\%$$

Some common conversions are

$$\frac{1}{10}=10\% \qquad \frac{1}{4}=25\% \qquad \frac{1}{2}=50\% \qquad \frac{3}{4}=75\%$$

Ratios are simply an alternative way of expressing fractions. Consider dividing £200 between two people in the ratio of 3:2. This means that for every £3 the first person gets, the second person gets £2. So the first gets $^3/_5$ of the total (i.e. £120) and the second gets $^2/_5$ (i.e. £80).

Generally, to split a quantity in the ratio $m : n$, the quantity is divided into $m/(m + n)$ and $n/(m + n)$ of the total.

Sigma Notation

The Greek capital letter sigma Σ is used as an abbreviation for a sum. Suppose we have n values: $x_1, x_2, \ldots x_n$ and we wish to add them together. The sum

$$x_1 + x_2 \ldots x_n \quad \text{is written} \quad \sum_{i=1}^{n} x_i$$

Note that i runs through all integers (whole numbers) from 1 to n. So, for instance

$$\sum_{i=1}^{3} x_i \text{ means } x_1 + x_2 + x_3$$

Example

$$\sum_{i=1}^{5} i^2 \text{ means } 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

This notation is often used in statistical calculations. The **mean** of the n quantities, $x_1, x_2, \ldots$ and x_n is

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

The variance is

$$\text{var}(x) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} x_i^2}{n} - \bar{x}^2$$

i.e. the mean of the squares minus the square of the mean

The **standard deviation** (sd) is the square root of the variance:

$$\text{sd}(x) = \sqrt{\text{var}(x)}$$

Note that the standard deviation is measured in the same units as x.

The Greek Alphabet

Α	α	alpha	Ι	ι	iota	Ρ	ρ	rho
Β	β	beta	Κ	κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda	Τ	τ	tau
Δ	δ	delta	Μ	μ	mu	Υ	υ	upsilon
Ε	ε	epsilon	Ν	ν	nu	Φ	φ	phi
Ζ	ζ	zeta	Ξ	ξ	xi	Χ	χ	chi
Η	η	eta	Ο	ο	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega