

INTRODUCTION TO  
NATURAL RESOURCE ECONOMICS

Lecture 3

**Non-renewable resource exploitation:  
externalities, exploration, scarcity  
and rents**

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## 1 Introduction

This lecture concludes our coverage of non-renewable resources by looking at externalities, exploration, resource scarcity and rent capture.

An appendix also presents an outline of the use of the *Hamiltonian* function in the analysis of dynamic optimisation problems. This will be useful when we turn our attention to renewable resource exploitation.

## 2 Externalities

External costs or *externalities* are costs that are incurred, in this case, in the process of extracting a natural resource, but which are not borne by the extracting firm itself. These costs may be incurred by other firms within the industry, by firms in other industries, or by individuals. They may affect only a relatively local area (around a mine, for example) or there may be a geographically more extensive effect. When the external costs of an activity are borne widely within society, we often talk about *social costs*. A good example of an externality in the context of non-renewable resources is the pollution of air or water by harmful substances used or released in the extraction process. In this case the costs imposed (in the form of impacts upon people's health, for example) cannot be measured directly since there is

(normally) no market for pollution or for good health. In general, the magnitude of such costs must be estimated using various *non-market valuation* techniques.

As a general principle, and provided that it is possible to measure (estimate) the relationship between, say, a pollutant and its harmful effect upon individuals, there is a trade-off between the benefits of allowing some pollution to occur (through cost savings in the extraction process) and the social costs of allowing that pollution. If we have full information about costs and benefits, we know that the optimal quantity of pollution is where the marginal benefit of allowing pollution to occur is just equal to the marginal (social) cost of pollution, i.e.,

$$MB(P) = MC(P).$$

Since it is usually the case that marginal benefits are decreasing in  $P$  while marginal costs are increasing in  $P$ , any more pollution would increase costs more than benefits but any less pollution would reduce benefits more than costs. Note that determining a socially optimal quantity of pollution does not imply that the costs and benefits of pollution are then distributed *fairly* between polluters and others within society!

Let  $z$  be a pollutant generated by extraction in proportion to the quantity extracted  $q$ . Assume that  $z \equiv \alpha q$  where  $\alpha$  is some positive number, but normalise  $\alpha$  to 1 so that we simply have  $z = q$ . A social planner would wish to modify a private firm's objective function to

$$\int_0^T [pq - c(q, x) - d(z, a) - \theta(a)] e^{-rt} dt,$$

where  $d(\cdot)$  is a *damage function* that gives the current social costs of the harm caused by the pollutant  $z$  and  $a$  represents *abatement* of the pollutant which can be achieved at a cost  $\theta(a)$ . The (current value) first order conditions of an appropriate Lagrangian with respect to  $q$  and  $a$  are now

$$p - c_q - d_z - \lambda = 0 \tag{1}$$

and

$$-d_a - \theta_a = 0. \tag{2}$$

Our rule for current extraction is now

$$p - c_q = \lambda + d_z, \tag{3}$$

so that, in order for extraction to be socially optimal, we need to take into account the current costs of extraction in terms not only of the impact on

profits in the future but also the marginal social costs of the resulting pollution. At the same time, we should observe the rule

$$\theta_a = -d_a, \tag{4}$$

which states that we should incur costs to abate (reduce) the level of pollution generated up to the point where the marginal cost of abatement equals the marginal reduction in the social costs of pollution.<sup>1</sup>

If we write  $d(\cdot)$  as  $d(z - a)$  then  $d_z = -d_a$  and we can put (3) and (4) together to obtain

$$p - c_q - \theta_a = \lambda,$$

which we could describe as the decision rule for a socially responsible firm (with perfect information). Assuming that  $d(\cdot)$  does not affect the firm directly, however, a profit-maximising firm clearly has an incentive to set  $\theta(a) = 0$ . How can we force the firm to incur abatement costs? We could use a pollution *tax* set at an appropriate level, we could impose *standards* to control the level of pollution, backed up with a system of inspections and penalties, or we could issue *permits* allowing a certain level of pollution which firms could trade between themselves in order, individually, to achieve the most efficient combination of permits and abatement for legal operation. In each case, the aim of the regulation is to make firms *internalise* the social costs of their pollution-generating activities.<sup>2</sup>

Note that we have here assumed that social costs are incurred only as a result of the (short run) effects of the *flow* of pollution. In some cases, if the pollutant is a long-lived substance, we also need to consider the effects of the *stock* of the pollutant. HSW (p.251) present an analysis that includes both flow and stock effects.<sup>3</sup>

### 3 Exploration

The search for new reserves is costly and, inevitably, subject to considerable uncertainty. Exploration, or prospecting, for new reserves incurs costs which must be deducted from the expected (net present) value of a reserve, which is itself uncertain. Firms therefore have an incentive to target areas with the lowest exploration costs first. New discoveries may not be developed immediately, but, depending upon extraction costs, extraction may be delayed until

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<sup>1</sup>If abatement is very costly, the firm may find it profitable instead to reduce the rate of extraction in order to reduce the level of pollution. Assume here that the price is high enough that it is always cost-effective to undertake abatement activities.

<sup>2</sup>This is a big topic in itself. Take a look at Chapters 3-5 in HSW.

<sup>3</sup>See also Chapter 6 of Conrad.

prices rise sufficiently (or technological advances reduce extraction costs). This is analogous to the idea of a backstop resource which only becomes available when prices reach a certain level.

Even the simplest analytical models of exploration are quite complex and are really beyond the scope of this course. For those interested, HSW present two alternative models of exploration in Section 8.4.

## 4 Scarcity

The notion of resource scarcity seems straightforward at first. But if we consider that a non-renewable resource may exist in plentiful quantities in the ground, but not necessarily in places and in concentrations where it is relatively easy to extract, we can begin to see that simple physical definitions of resource scarcity are perhaps not very useful. In addition, a resource cannot really be said to be scarce if there is little or no demand for it at a price at which someone would find it profitable to extract and supply it.

An economic definition of scarcity therefore needs to take into account prices and costs. Broadly speaking, from an economic perspective a resource becomes more scarce, all else equal, if demand (and hence the market price) increases and less scarce if demand decreases, perhaps because of the appearance of close substitutes for the resource. What is less intuitive is that economic scarcity can be said to *decrease* if the costs of extraction increase. If extraction costs are rising faster than prices, then the implication is that society (the market) does not regard the resource as more scarce.

Recall that, along an efficient extraction path, the *shadow price* or value  $\lambda$  of the resource stock equals the difference between price and marginal costs:

$$\lambda = p - c_q.$$

The shadow price is usually regarded as the correct measure of economic scarcity. Remember that the shadow price depends upon prices and costs throughout the lifetime of the resource (including the backstop price), whereas price and marginal cost are current values. A *trend* in the shadow price, rather than a point value, is the best indicator of resource scarcity. If the shadow price is increasing, the resource is, in economic terms, becoming more scarce. Because the shadow price depends upon both prices *and* costs, changes in prices or costs alone are difficult to interpret as indicators of resource scarcity. For example, since

$$\dot{p} = \dot{\lambda} + \dot{c}_q,$$

depending on the value of  $\dot{c}_q$ , it is clearly possible for  $\dot{p}$  and  $\dot{\lambda}$  to have opposite signs.

## 5 Rent capture

Natural resources can be very valuable. The significant economic profits which may be generated through the extraction and sale of a resource stock can be thought of as *resource rent*, i.e., an economic rent that is attributable to the resource itself. Government may take the view that resource rents should, in principle, accrue to the State to be used for the benefit of society as a whole, rather than be allowed to remain in private hands. It is therefore common for Governments to make charges on resource-owning companies in an effort to *capture* all or part of the resource rent.

Two common methods of charging are a revenue tax (or, equivalently, a tax on the quantity extracted) and a profit tax (sometimes referred to as a *royalty*).

A revenue (or *severance*) tax, in effect, lowers the market price that the firm receives. With a unit tax  $\tau$  on its production, a competitive firm seeks to maximise

$$\int_0^T [(p - \tau)q - c(q, x)] e^{-rt} dt,$$

or, with an *ad valorem* tax,

$$\int_0^T [[1 - \tau]pq - c(q, x)] e^{-rt} dt.$$

In either case, the effect is similar to that of an increase in the costs of extraction; for example, a flat-rate tax on production is equivalent to a constant marginal cost of production. With a revenue tax, we would expect the resource owner to cease extraction earlier and to leave more of the resource in the ground. Because the profit-maximising behaviour of the firm is affected by the tax, we can say that the tax is *distortionary*.

A profits tax or royalty levied at a rate  $\tau$  changes the firm's objective function to

$$\int_0^T [1 - \tau] [pq - c(q, x)] e^{-rt} dt = [1 - \tau] \int_0^T [pq - c(q, x)] e^{-rt} dt.$$

Since we can divide through the entire integral function by  $[1 - \tau]$  and leave it unaltered, it is apparent that the tax is not going to change the profit-maximising behaviour of the firm and it is therefore not distortionary.

A profits tax does, however, clearly reduce the present value of the resource. It may, therefore, reduce incentives for companies to explore or prospect for new reserves.

In some cases, resource stocks may already be in public ownership. The Government is then in a position to sell the rights to extract the resource.

Some form of competitive tendering or bidding process could then be used in order to capture the (discounted) resource rent at the outset. What would the maximum amount be that a firm would be willing to pay now for the right to extract a non-renewable resource over its economic lifetime?

## 6 Further reading

Look at Conrad, pp.91-100, and HSW, Chapter 8.

## Appendix: the Hamiltonian function

We said in an earlier lecture that the conditions for the efficient exploitation of a natural resource could be obtained directly if we set up the Lagrangian for the resource owner's (social planner's) problem in a particular way, called a *Hamiltonian*. Below we look, briefly, at the derivation of the Hamiltonian for a simple resource problem and the conditions for an optimal solution.

Define an instantaneous profit function

$$\pi(q(t), x(t)) \equiv pq(t) - c(q(t), x(t)),$$

with  $\pi_{qq} \equiv -c_{qq} < 0$  and  $\pi_x \equiv -c_x \geq 0$ . This just states that profits are increasing in  $q$  (but at a decreasing rate) and may also be increasing in  $x$ , the size of the resource stock.

The (social planner's) problem is to

$$\begin{aligned} \max_q \int_0^T \pi(\cdot) e^{-rt} dt, \\ \text{s.t. } \dot{x} = f(x(t), q(t)), \quad x(0) = x_0. \end{aligned}$$

The constraints here state that the size of the stock is evolving according to a function  $f(\cdot)$  of the stock itself (if there is *growth*) and the extraction rate  $q(t)$ , and that the initial size of the stock is given (as  $x_0$ ).

Define a Lagrangian function in the usual way as

$$\mathcal{L} \equiv \int_0^T \{ \pi(\cdot) e^{-rt} + \mu(t) [f(\cdot) - \dot{x}] \} dt, \quad (5)$$

noting that here the multiplier  $\mu$  implicitly denotes a *discounted* shadow price. Thus

$$\mu(t) \equiv \lambda(t) e^{-rt},$$

where  $\lambda$  is the *current value* shadow price.

Now, define a new function  $\mathcal{H}(\cdot)$  where

$$\mathcal{H}(\cdot) \equiv \pi(\cdot) e^{-rt} + \mu(t) f(\cdot),$$

so that

$$\mathcal{L} = \int_0^T \mathcal{H}(q, x, \mu, t) dt - \int_0^T \mu(t) \dot{x} dt. \quad (6)$$

The function  $\mathcal{H}(\cdot)$  is the (present value) *Hamiltonian*.

If we integrate the term in the second integral in (6) *by parts* (look at any good “mathematics for economists” text) to obtain

$$\int_0^T \mu(t) \dot{x} dt = \mu(T) x(T) - \mu(0) x_0 - \int_0^T \dot{\mu} x(t) dt,$$

we can rewrite the Lagrangian as

$$\mathcal{L} \equiv \int_0^T \{\mathcal{H}(\cdot) + \dot{\mu} x\} dt + \mu(0) x_0 - \mu(T) x(T). \quad (7)$$

Differentiating (7) with respect to  $q$ ,  $x$  and  $\mu$  gives the first order necessary conditions

$$\mathcal{L}_q = \mathcal{H}_q = \pi_q e^{-rt} + \mu f_q = 0, \quad (8)$$

$$\mathcal{L}_x = \mathcal{H}_x + \dot{\mu} = \pi_x e^{-rt} + \mu f_x + \dot{\mu} = 0 \quad (9)$$

and, after substituting  $\int_0^T \mu(t) \dot{x} dt$  back into (7),

$$\mathcal{L}_\mu = \mathcal{H}_\mu - \dot{x} = f(\cdot) - \dot{x} = 0. \quad (10)$$

From (8), optimal exploitation requires

$$\pi_q e^{-rt} = -\mu f_q$$

or

$$\pi_q e^{-rt} = \mu, \quad (11)$$

if, as is usually the case,  $f(\cdot) \equiv g(x) - q$  (where  $g(x)$  is a growth function) and hence  $f_q = -1$ . This is equivalent to the familiar *current period* condition for optimal extraction

$$\pi_q = \lambda. \quad (12)$$

From (9) we get what is known as the *costate condition*<sup>4</sup> or *adjoint equation*

$$\dot{\mu} = -\pi_x e^{-rt} - \mu f_x \quad (13)$$

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<sup>4</sup>Here,  $q$  is the so-called *control* variable and  $x$  is the *state* variable.

which describes the change in the (discounted) shadow price of the stock over time. Note that, from  $\mu(t) \equiv \lambda(t) e^{-rt}$  we can find

$$\dot{\mu} = \left[ \dot{\lambda} - r\lambda \right] e^{-rt}$$

and hence, in current value terms, the costate condition becomes

$$\dot{\lambda} = -\pi_x - \lambda[f_x - r]. \quad (14)$$

Notice that for a non-renewable resource, where  $f_x = g'(x) = 0$ , this condition becomes more familiar as

$$\dot{\lambda} = \lambda r - \pi_x,$$

with resource-dependent costs, and

$$\dot{\lambda} = \lambda r$$

without.

This is the bare bones of a methodology for solving dynamic optimisation problems known as *optimal control theory*. See HSW, Box 7.2 or look at Conrad (Section 1.2) for a simplified discrete-time version.

So, what exactly does the Hamiltonian represent? The present value Hamiltonian is

$$\mathcal{H}(\bullet) \equiv \pi(\bullet) e^{-rt} + \mu(t) f(\bullet),$$

while its current value equivalent can be written as

$$\mathcal{H}^c(\bullet) \equiv \pi(\bullet) + \lambda(t) f(\bullet),$$

where  $f(\bullet)$  is the function that describes the (instantaneous) change in the size of the resource stock. The Hamiltonian measures the total rate of increase in the (current or present) *value* of the managed resource. Here,  $\pi(\bullet)$  is the net flow of returns from the resource and  $\lambda(t) f(\bullet)$  is the increase in the value of the resource stock, taking into account both extraction and growth (if any).