Last class we looked at the axioms of expected utility, which defined a rational agent as proposed by von Neumann and Morgenstern.

We then proceeded to look at empirical evidence of violations of each of the axioms.

It seems clear that, although important in a normative sense, expected utility fails to describe human behavior well.

So, we will look at theories of behavior that attempt to capture behavior.
Let's remember some notation before we can proceed.

A **prospect** $(x_1, p_1; \ldots; x_n, p_n)$ is a contract that yields outcome $x_i$ with probability $p_i$, where $p_1 + p_2 + \ldots + p_n = 1$.

For example, the gamble where I win £1 if heads comes out of a flip of a coin and where I lose £1 if tails comes out would be expressed as:

$(1, 1/2; -1, 1/2)$
There are three basic principles economists use when applying EUT

(1) **Expectation** \( u(x_1, p_1; \ldots; x_n, p_n) = p_1 u(x_1) + \ldots + p_n u(x_n) \)

(2) **Asset Integration** \((x_1, p_1; \ldots; x_n, p_n)\) is acceptable at wealth level \(w\) if and only if \(u(w + x_1, p_1; \ldots; w + x_n, p_n)\) (In other words, the domain of utility is final wealth level, not gains or losses)

(3) **Risk Aversion** \(u\) is concave \((u'' < 0)\)
Kahneman and Tversky (1979) proposed a theory that addressed key shortcomings of EUT:

Certainty Effect

Losses versus Gains
Prospect Theory distinguishes 2 phases in the choice process:

- Editing
- Evaluation

Editing phase is a preliminary analysis of the problem,

- it works as a simplification of the problem through basic operations
Prospect Theory: Editing Phase Operations

Coding

- Determining what the reference point is
- This in turn helps clarify what is a gain and what is a loss

Combination

- Combining probabilities of identical outcomes
- E.g. \((200, .25; 200, .25) = (200, .5)\)
Prospect Theory: Editing Phase Operations

**Segregation**
- Separating riskless component from risky components;
- E.g. \((300, .80; 200, .20)\) is \((200) + (300, .80)\)

**Cancellation**
- Elimination of components which are common to two gambles
- E.g. \(A = (200, .20; 100, .50; -100, .30)\) vs \(B = (200, .20; 150, .50; -100, .30)\)
- A and B can be simplified to:
- E.g. \(A = (100, .50; -100, .30)\) vs \(B = (150, .50; -100, .30)\)
Prospect Theory: Evaluation

Once editing is complete, individuals evaluate the prospects and choose that of highest value.

The value of a prospect will be given by $V$, $V$ in turn depends on two scales: $\pi$ and $\nu$.

$\pi$ associates a decision weight $\pi(p)$ to a probability $p$.

However, $\pi(p)$ is not a probability!
Prospect Theory: Evaluation

The second scale, $\nu$ assigns to each outcome $x$ a number $\nu(x)$ which reflects the subjective value of that outcome.

Remember that outcomes are measured as deviations from a reference point

$\nu$ measures the value of such deviations
Prospect Theory: Evaluation

We’re going to work with a simple formulation of prospects: 

$$(x, p; y, q)$$

In this class of prospects, one gets:

- $x$ with probability $p$
- $y$ with probability $q$
- $0$ with probability $1 - p - q$
Prospect Theory: Evaluation

A prospect is *Strictly Positive* if:
- \( x, y > 0 \) and
- \( p + q = 1 \)

A prospect is *Strictly Negative* if:
- \( x, y < 0 \) and
- \( p + q = 1 \)

Otherwise, we have a *Regular Prospect*
Regular Prospects are evaluated following this equation:

\[ V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y) \]

\( v(0) = 0, \pi(0) = 0 \) and \( \pi(1) = 1 \).

\( V \) is defined on prospects, while \( v \) is defined on outcomes.

- \( V \) and \( v \) only coincide for sure prospects
- \( V(x, 1) = V(x) = v(x) \)
Prospect Theory: Evaluation of Strict Prospects

Strict prospects are evaluated differently

In the editing phase, they are divided in two components:

▶ The sure component
▶ The risky component

\[ V(x, p; y, q) = \pi(p)v(y) + [1 - \pi(p)]v(y) \]

which can be re-written as:

\[ V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)] \]
Prospect Theory: Evaluation of Strict Prospects

\[ V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)] \]

That is, the value of the strict prospect is the value of the sure component plus the difference between sure and risky components, multiplied by the decision weight associated with the more extreme outcome.

That is, the value of the strict prospect is the value of the sure component plus the difference between sure and risky components, multiplied by the decision weight associated with the more extreme outcome.
The key notion in the value function is that it depends on two main factors:

The reference point and changes relative to it.

Psychologically, it is intuitive that we respond to changes from a given point rather than to absolute values.

Furthermore, typically people are more averse to losses than to gains.

How much would you pay NOT to play the following gamble?

$\left(-10, \frac{1}{2}; 10, \frac{1}{2}\right)$
In fact, would that value differ if the lottery was this?

\[ (-100, \frac{1}{2}; 100, \frac{1}{2}) \]

This means that the value function is steeper for losses than for gains.

The sensitivity to a loss or gain is highest near the reference point.
Prospect Theory: The Value Function

valuef.jpg
Prospect Theory: The Value Function Overview

The Value function $V(X)$, where $X$ is a prospect:

Is defined by gains and losses from a reference point

Is concave for gains, and convex for losses
  - The value function is steepest near the point of reference:
    - Sensitivity to losses or gains is maximal in the very first unit of gain or loss

Is steeper in the losses domain than in the gains domain
  - Suggests a basic human mechanism (it is easier to make people unhappy than happy)
  - Thus, the negative effect of a loss is larger than the positive effect of a gain
In Prospect Theory, the value of each outcome is multiplied by a decision weight, $\pi(p)$.

Nevertheless, decision weights have certain desirable properties:

- $\pi(0) = 0$
- $\pi(1) = 1$

Hence, impossible events are ignored and the scale is normalized.
Does this mean that $\pi(p)$ is linear?

**Problem A**

$(5,000, .001)$ $(5, 1)$  

$N=72$ $[72\%]$ $[28\%]$  

**Problem B**

$(-5,000, .001)$ $(-5, 1)$  

$N=72$ $[17\%]$ $[83\%]$
Under gains the lottery is preferred to the sure outcome:

\[ \pi(0.001)v(5,000) > v(5) \iff \pi(0.001) > \frac{v(5)}{v(5,000)} \]

\[ \frac{v(5)}{v(5,000)} > 0.001 \text{ if } v(x) \text{ is concave} \]
Note that the overweighing of low probabilities is not the same as overestimation

Here probabilities are explicitly given, unlike in real world.

If anything, the two effects may work together.
Consider the following choices:

Choice 1:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th></th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>$</td>
<td>Probability</td>
<td>$</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>0.1</td>
<td>500</td>
</tr>
<tr>
<td>0.89</td>
<td>100</td>
<td>0.01</td>
<td>0</td>
</tr>
</tbody>
</table>
### Allais Paradox Revisited

**Choice 2:**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Probability</strong></td>
<td><strong>$</strong></td>
</tr>
<tr>
<td>0.1</td>
<td>500</td>
</tr>
<tr>
<td>0.9</td>
<td>0</td>
</tr>
</tbody>
</table>
Choosing A implies:
\[ v(100) > \pi(0.1)v(500) + \pi(0.89)v(100) \]

\[ \iff (1 - \pi(0.89))v(100) > \pi(0.1)v(500) \]

Choosing B implies:
\[ \pi(0.1)v(500) > \pi(0.11)v(100) \]

Combining the two inequalities, it means that
\[ (1 - \pi(0.89))v(100) > \pi(0.11)v(100) \text{ or } \]
\[ \pi(0.89) + \pi(0.11) < 1 \]
Prospect Theory: The Weighting Function

In short, the weighing function can be characterized by:

Overweighing: It will give more weight to low probability outcomes.

Subadditivity: decision weights need not add up to one.
Prospect Theory: The Weighting Function
Imagine the following situation:

Situation A:
You are about to purchase a jacket for £125 and a calculator for £15. The salesman mentions that the calculator is on sale for £10 at another branch of the store 20 minutes away by car. Would you make the trip?

Situation B:
You are about to purchase a calculator for £125 and a jacket for £15. The salesman mentions that the calculator is on sale for £120 at another branch of the store 20 minutes away by car. Would you make the trip?

68% (N=88) of subjects were willing to drive to the other store in A, but only 29% (N=93) in B.
Kahneman and Tversky (1984) propose three types of mental accounts:

**Minimal:** Examining options by looking only at the differences between them, disregarding any commonalities.

**Topical:** Relating the consequences of possible choices to a reference level that is determined by the context within which the decision arises.

**Comprehensive:** Incorporating all other factors and all available information, like current wealth, future earnings etc.
Let’s see how each type of account would handle this problem:

**Minimal**: decision-maker only considers differences between local options.
- do I drive 20 minutes to save £5?
- answer is the same in both problems


**Comprehensive:** d-m considers all relevant information including wealth

- Let \( W \) be current wealth and

- \( W^* \) be wealth + calculator + jacket - £140

- d-m has to decide between \( W^* + 20 \) minutes and \( W^* - £5 \)

- answer is the same in both problems!
Mental Accounting: our example revisited

**Topical:** d-m considers the context in which the decision arises

- reducing the price of the calculator from £15 to £10
- or reducing the price of the calculator from £125 to £120
- discount is more salient when the calculator costs £15

\[ v(-125) - v(-120) < v(-15) - v(-5) \]

- this follows from the convexity of the value function in the loss domain.
The late Paul Samuelson proposed the following famous problem:

Having lunch with a colleague, he offered him the following bet:

- They would flip a coin
- If the colleague won, Samuelson would pay him $200
- If the colleague lost, Samuelson would get $100 from him
His colleague promptly rejected the offer. His reasoning was:

‘I would feel the $100 loss more than the $200 gain.’

However, he said that if Samuelson would be willing to play this 100 times, he would be game.
Samuelson showed that this is irrational:

If you reject one flip you should also reject a sequence of two flips.

But after seeing the first flip, you will reject the second, because you dislike playing a single flip!

Hence, you should also reject a sequence of 3 flips, and so on.
From a behavioral perspective, two things are noteworthy

1) ‘I would feel the $100 loss more than the $200 gain.’ (i.e. I am loss averse)

2) I’ll play a sequence of flips rather than 1 flip
If each coin flip is handled as a separate event, then 2 flips are twice as bad as one.

What if the two bets are combined into one portfolio?

The gamble becomes: ($400, .25; $100, .50; −$200, 0.25)

This is now acceptable (either if you are risk neutral or loss averse)

- Hence, Samuelson’s colleague should accept the series of coin flips but not watch them unfold!
You may argue/think risk aversion could explain this.

Suppose Samuelson’s colleague has

- a ‘standard’ utility function $U(x) = \ln x$ and
- wealth of $10,000$

What is the $x$ which makes him indifferent between playing this lottery or not?

($x, .5; -100, .5$)
What is the $x$ which makes him indifferent between playing this lottery or not?

($x, .5; −$100, .5)

$x = 101.01!$
Rabin (1998) shows that someone who turns down Samuelson’s gamble should also turn down the following gamble:

- 50% chance of losing $200
- 50% chance of winning $20,000

Rabin shows that expected utility theory requires people to be risk neutral when stakes are low.

To explain such behavior, one requires a combination of

- loss aversion;
- one-bet-a-time mental accounting
The equity premium puzzle is the empirical fact that returns on stocks are higher than bonds.

Benartzi and Thaler (1995) report that stocks outperformed bonds by 6%.

- $1 invested in stocks in 01/01/1926 would be worth more than $1800 in 01/01/1998
- $1 invested in Treasury bills in 01/01/1926 would be worth more than $15 in 01/01/1998

The puzzle comes from the fact that the risk aversion necessary to explain this phenomenon is implausible.

- the CRRA required would be 40
Benartzi and Thaler (1995) analyse what a loss averse fund manager/investor would behave if

- his performance is evaluated regularly
- he evaluated his position regularly

This in effect is equivalent to the d-m re-setting his/her reference point.

In particular, what is the frequency of evaluation which makes investors indifferent between historical distributions of returns on stocks and bonds?
In particular, what is the frequency of evaluation which makes investors indifferent between historical distributions of returns on stocks and bonds?

Answer: 13 months!

1 year is a very plausible time-frame which investors’ performance is evaluated.

As such, the equity-premium could therefore be a function of *Myopic Loss Aversion*
Myopic Loss Aversion & Narrow Framing

Myopic Loss Aversion is an example of Narrow Framing

Projects are evaluated one at a time, rather than as a part of an overall portfolio

Camerer et al. (1997) study the decision-making of NYC taxi drivers.

In NY, taxi drivers rent their cars for 12 hours for a fixed fee.
   • They keep all the money they make during that period

The key decision is how long to work on a given day.
Some days are busier than others

A rational cab driver should work longer on busy days and less on slow days
  ▶ This maximises per-hour wage

Instead, drivers establish a daily earnings target and quit early on busy days.

Taxi drivers seem to do their mental accounting on a daily basis.
The 4 fundamental principles in Behavioral Economics

1) Outcomes are evaluated as changes around a reference point.

2) Losses loom larger than gains

3) Probabilities are not weighed linearly
   - Rare events are overweighed
   - Very frequent events are underweighted
   - There is a discontinuity from certainty to probability

4) Decision-making is done via mental accounts.