Recap

Last week we looked at how individuals process new information to update their beliefs about a given process Bayes rule.

We also looked at a very simple model of search

We saw the implications of different costs of searching for

- length of search and
- optimal reservation value.
Today we are going to have a look at behaviour in strategic settings.

We will look at the fundamental building block of non-cooperative game theory, the Nash equilibrium.

We will focus our attention at games where there is more than one equilibrium.
A game is defined by three basic elements:

- A set of players: $N$
- A set of strategies: $S$
- A rule (or function), $F$, that maps strategies to outcomes.

Game theory's objective is to analyse the games outcome given a set of preferences.

Traditionally, game theorists assume agents seek to maximise their own payoffs more on this later.
In particular, game theory is interested in finding outcomes from which players have no incentive to deviate.

- i.e. outcomes in which my actions are optimal given what the other players are doing (and vice versa).

Such an outcome is a Nash Equilibrium (NE) of a game.

Some games have unique NE; others have many NE.
Example 1

Take two workers operating in a factory.

Their payoff is a function of joint output.

However each worker has a private cost of effort.
Example 1

$N = \{1, 2\}$

$S_i = \{High, low\}$

<table>
<thead>
<tr>
<th></th>
<th>Player 1 High</th>
<th>Player 2</th>
<th></th>
<th>Player 1 Low</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>20, 20</td>
<td>5, 15</td>
<td>Low</td>
<td>15, 5</td>
<td>10, 10</td>
</tr>
<tr>
<td>Low</td>
<td>15, 5</td>
<td>10, 10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 1

There are two Nash equilibria in pure strategies:

- (High, High)
- (Low, Low)

In this game, strategies are strategic complements:

Player 2's best response to a rise (drop) in player 1's action is a rise (drop) in his action.

Which equilibrium should be played?
Coordination Games

This particular type of game is interesting to economists as it captures the idea of externalities:

- Team production processes (e.g. min. effort game);
- Industrial Organisation (e.g. market entry games);

It is important to understand why would a set of agents be stuck in bad equilibria.

Is this due to strategic or behavioural reasons?
Equilibrium selection in games

Common criteria for equilibrium selection:

Focal points;

Payoff dominance;

Risk dominance.
Thomas Schelling proposed a class exercise to his students. They had to select a time and a place to meet up in New York city the following day.

The majority of his students chose Grand Central Station at 12 noon.

Certain equilibria are “intuitive” or naturally salient and as a result get chosen more often.
This paper studies the extent to which salience of decision labels could lead to resolution of the coordination problem.

In pilot data, they modified Schelling’s example and set up a simple coordination game

University of Chicago students had to choose to meet in one of two locations:

- The Sears Tower, a landmark Chicago building;
- The AT&T Tower, a little known building across the street from the Sears Tower.

They considered three conditions
Symmetric Treatment:

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Sears Tower</th>
<th>AT&amp;T T Tower</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sears Tower</td>
<td>100, 100</td>
<td>0, 0</td>
</tr>
<tr>
<td>AT&amp;T Tower</td>
<td>0, 0</td>
<td>100, 100</td>
</tr>
</tbody>
</table>
Slightly Asymmetric Treatment:

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Sears Tower</th>
<th>AT&amp;T Tower</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sears Tower</td>
<td>101, 100</td>
<td>0, 0</td>
</tr>
<tr>
<td>AT&amp;T Tower</td>
<td>0, 0</td>
<td>100, 101</td>
</tr>
</tbody>
</table>

Focal points: Crawford et al. (2008)
Asymmetric Treatment:

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Sears Tower</th>
<th>AT&amp;T Tower</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sears Tower</td>
<td>110, 100</td>
<td>0, 0</td>
</tr>
<tr>
<td>AT&amp;T Tower</td>
<td>0, 0</td>
<td>100, 110</td>
</tr>
</tbody>
</table>
The percentage of subjects who chose “Sears Tower” is as follows:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>High Payoff</th>
<th>Low Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry</td>
<td>90% (n=60)</td>
<td></td>
</tr>
<tr>
<td>Slight Asymmetry</td>
<td>58% (n=50)</td>
<td>61% (n=49)</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>47% (n=30)</td>
<td>50% (n=28)</td>
</tr>
</tbody>
</table>
Expected coordination rates were equal to:

- Symmetry: 82%
- Slight Asymmetry: 52%
- Asymmetry: 50%
- Mixed Strategy Nash Equilibrium: 50%!

The mere presence of small payoff asymmetries dramatically reduces the power of focal points (in Crawford et al.'s data set).
Payoff Dominance

Payoff dominance is a relatively intuitive concept;

If an equilibrium is Pareto superior to all other NE, then it is payoff dominant.

An outcome Pareto-dominates another if all players are at least as well off and at least one is strictly better off.

It is intuitively appealing, but the data does not seem to fully support it.
The concept of risk dominance is based upon the idea that a particular equilibrium may be riskier than another.

- Though players need not be risk averse.

In simple 2x2 games, RD could be thought as how costly are deviations from a particular equilibrium vis--vis the other?

- There is no general way to compute a risk dominant equilibrium in $n \times n$ games.

Although rational agents ought to follow payoff dominance, experimental data shows subjects often play the risk dominant (“safer”) equilibrium.
Obstacles to coordination

Larger N

The concept of coordination centres around beliefs.

The choice of equilibrium will depend on what you think the other player will do.

The more players there are, the harder it is to coordinate: it is harder to form consistent beliefs about every players action. It only takes one player to destroy the equilibrium.
Obstacles to coordination

Incentive structure

Are equilibria unfair?

Are there focal points?

Does communication help?
Cooper et al. (1992)

Run a simple coordination experiment.

Vary the extent subjects can communicate with one another:

- No communication;
- One-way (non-binding) announcements;
- Two-way (non-binding) announcements.
Cooper et al. (1992)

<table>
<thead>
<tr>
<th>Row Player's Strategy</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>800,800</td>
<td>800,0</td>
</tr>
<tr>
<td>2</td>
<td>0,800</td>
<td>1000,1000</td>
</tr>
</tbody>
</table>

**Figure II**
<table>
<thead>
<tr>
<th>Announcements</th>
<th>Strategy</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No communication:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-way:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep. 1 &amp; 3</td>
<td>19</td>
<td></td>
<td></td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>Rep. 2</td>
<td>2</td>
<td></td>
<td></td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>21</td>
<td></td>
<td></td>
<td>144</td>
<td></td>
</tr>
<tr>
<td>Two-way:</td>
<td>0</td>
<td></td>
<td></td>
<td>330</td>
<td></td>
</tr>
<tr>
<td>Actions:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No communication:</td>
<td></td>
<td>325</td>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>One-way:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep. 1 &amp; 3</td>
<td>88</td>
<td></td>
<td></td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>Rep. 2</td>
<td>15</td>
<td></td>
<td></td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>103</td>
<td></td>
<td></td>
<td>227</td>
<td></td>
</tr>
<tr>
<td>Two-way:</td>
<td>15</td>
<td></td>
<td></td>
<td>315</td>
<td></td>
</tr>
</tbody>
</table>

| Treatment:                    |          |       |     |       |       |
|                               | (1,1)    | (2,2) | (1,2), (2,1) |
| No communication:             |          |       |     |       |       |
| One-way:                      |          |       |     |       |       |
| Rep. 1 & 3                    | 25       | 47    |     | 38    |       |
| Rep. 2                        | 1        | 41    |     | 13    |       |
| Total                         | 26       | 88    |     | 51    |       |
| Two-way:                      | 0        | 150   |     | 15    |       |
Example 2

Consider a firm with two workers.

- Their payoff depends on the total output produced
- That in turn depends on worker effort, which is costly

How would we model this?
Example 2: Team Production

\[ N = \{1, 2\} \]

\[ S_i = \{\text{High, low}\} \]

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>80, 80</td>
<td>40, 90</td>
</tr>
<tr>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>90, 40</td>
<td>50, 50</td>
</tr>
</tbody>
</table>
Example 2: Team Production

The equilibrium is found where players are picking a strategy which maximises their profits, given what their counterparts are doing.

- (Low, Low)

The optimal solution for the firm is for workers to exert high effort.

- Firm output is maximised;
- Payoffs for workers are maximised.

However, the dominant strategy is to shirk

- How can this be overcome?
A solution is to hire a monitor to ensure workers exert high effort.

Solution is cost effective as long as the cost of monitoring is the extra revenue from high effort.

But who monitors the monitor?

▶ The monitor also has an incentive to shirk!!

The only monitor without such incentives is the owner of the firm.
Performance-related pay

Piece rates. In other words, workers get $w$ for each unit ($q$) they produce.

This type of contract goes back to Taylor in the XIX century; it is still widely used in the agricultural sector.

Individuals will work until $MC(q) = w$. 
Alternative contractual solutions

However, how does one set $w$?

If $w$ is set based on previous performance, there is a moral hazard problem: workers have an incentive to underperform.

Also, the applicability of piece rates is limited to agricultural or industrial contexts.
Another possibility is to set a fixed target to a team.

- If achieved, bonus is shared by the group.
- If not, each group member is paid a basic wage, which is typically low (unless you are an investment banker).

Target-based schemes are very popular in the services industry (e.g. retail, inv. banking).
How do these contracts compare?

Bandiera et al. (2006) compare piece rates to a productivity-based contract

- wage = $\beta K$,
- $K$ is kilos of picked fruit by worker
- $\beta = w/y$
- $w = \text{minimum wage} + \text{constant}$
- $y = \text{mean daily productivity of group}$
The relative performance contract was introduced to control for productivity shocks

- e.g. bad weather in one part of the farm means those workers will earn less

Under this contract working hard implies (all else constant):

- Higher earnings ($K \uparrow$)
- Higher average effort and thus higher average productivity ($y \uparrow$)
- This in turn leads to lower earnings for everyone else

This contract has a PG aspect to it – what is good for an individual is detrimental to other group members.
Farm workers were temporary workers from outside the UK.

Productivity under the piece rate was 50% higher than under the relative incentive scheme.

The latter leads to creation of social norms of cooperation among co-workers (i.e. lower effort).
The fact that workers come from different parts of Eastern Europe provides a source of heterogeneity:

- Countries have different living standards, hence the same piece rate gives different income to each nationality.

Bandiera et al. (2006) find (perhaps unsurprisingly) that the larger the value of the piece rate (as a function of average salary in home country), the higher the productivity of the worker.
Another alternative is to pay workers based on their relative performance:

- Promotion Tournaments.

These contracts work much like sports competitions:

- The individual who is more productive wins either a bonus or a promotion.

- A variant of this type of contract was in place at GE under their former CEO, Jack Welsh. Every year, the bottom 10% managers would be sacked!
Tournaments

Consider a firm with two employees.

Their contract specifies a bonus, whose size depends on who is most productive.

- $M$ for the most productive;
- $m$ for the least productive.

Individuals like getting paid, but don't like working;

The boss cannot perfectly observe effort. Hence your output is a function of effort plus luck:

- $Y_i = f(e_i) + \varepsilon_i$
Assume $\varepsilon$ is between $[-a, a]$, where $a > 0$

- you can be unlucky and get a negative $\varepsilon$ or
- you can be lucky and get a positive $\varepsilon$

Effort is between 0 and 100 and cannot be recovered whether you win or lose.
So, if player 1 and player 2 put in effort levels $e_1$ and $e_2$, player 1s expected utility of playing this contest is:

$$EU_1(e_1, e_2) = p(e_1, e_2)U(M) + [1 - p(e_1, e_2)]U(m) - c(e_1)$$
You can show the Nash equilibrium only depends on two elements:

- $M$ and $m$: the incentive structure
- $a$: the degree to which the company correctly identifies effort

If $a$ is distributed uniformly, the equilibrium effort level is given by the following equation:

$$e^* = \frac{(M-m)c}{4a}$$
Bull, Schotter and Weigelt (1987) compare tournaments to piece rates in controlled experiments.

They set up a series of tournaments in which the predicted equilibrium effort is equal to that of a piece rate.

They find that on average, subjects' effort levels were consistent with theory.

However, piece rates have a much lower variance in effort levels

▶ A piece rate is a simpler problem w/out strategic uncertainty.
Tournaments

Mueller and Schotter (2003) study tournaments where they manipulate individual subject ability

- A low ability subject must put more effort to have an equal chance of winning.

They find that:

- High ability subjects work harder than predicted;
- Low ability subjects simply drop out.

So, relative performance mechanisms may lead to dropout/workaholic behaviour.

Even if total output is higher, it is unclear whether it is desirable to have such a corporate culture.
Is on-the-job competition always healthy?

- We've all heard of certain work environments being cut-throat is this due to incentives?

Harbring and Irlenbusch (2005) study sabotage in contests.

- Subjects can spend resources in order to reduce their counterparts probability of winning the contest.
Harbring and Irlenbusch (2005) vary
  ▶ the number of $M$ and $m$ prizes on offer
  ▶ Number of players in the contest: 2, 4 and 8.

The number of competitors does not seem to affect average effort.

However, the ratio $M/m$ seems to have an impact on effort, but not on sabotage efforts.
### Table 5: Overview of experimental results regarding effort costs and sabotage costs

<table>
<thead>
<tr>
<th>tournament size $n$</th>
<th>1m1M</th>
<th>3m1M</th>
<th>2m2M</th>
<th>1m3M</th>
<th>6m2M</th>
<th>4m4M</th>
<th>2m6M</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 agents</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fraction of winner prizes $\lambda$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>average cost of effort</td>
<td>32.92</td>
<td>23.10</td>
<td>34.71</td>
<td>24.76</td>
<td>22.86</td>
<td>31.45</td>
<td>23.72</td>
</tr>
<tr>
<td>average cost of sabotage</td>
<td>19.75</td>
<td>18.01</td>
<td>12.30</td>
<td>12.63</td>
<td>13.58</td>
<td>17.51</td>
<td>12.70</td>
</tr>
<tr>
<td>cost of sabotage in equilibrium</td>
<td>3.33</td>
<td>3.33</td>
<td>3.33</td>
<td>3.33</td>
<td>3.33</td>
<td>3.33</td>
<td>3.33</td>
</tr>
<tr>
<td>median ratio of costs of effort and sabotage</td>
<td>1.73</td>
<td>1.43</td>
<td>2.39</td>
<td>1.91</td>
<td>1.16</td>
<td>2.18</td>
<td>2.25</td>
</tr>
<tr>
<td>average ratio of marginal costs of effort and sabotage</td>
<td>0.73</td>
<td>0.64</td>
<td>0.94</td>
<td>0.79</td>
<td>0.71</td>
<td>0.75</td>
<td>0.77</td>
</tr>
</tbody>
</table>
So far, we've studied tournaments in a somewhat abstract framework.

▶ Is this really realistic?

Freeman and Gelber (2006) study tournaments in which effort is done in real terms.

▶ Subjects are asked to solve puzzles

How does information about skill affect outcomes?

How does inequality in rewards \((M - m)\) affect outcomes?
Table 1 *Means and Standard Deviations of Mazes Solved per person, by Treatment*

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(2) Information</th>
<th>(3) Total Participants</th>
<th>(4) Round 1</th>
<th>(5) Round 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Inequality</td>
<td>Full</td>
<td>72</td>
<td>12.69 (5.04)</td>
<td>15.79 (7.26)</td>
</tr>
<tr>
<td>Medium Inequality</td>
<td>Full</td>
<td>66</td>
<td>12.50 (4.49)</td>
<td>18.76 (7.29)</td>
</tr>
<tr>
<td>High Inequality</td>
<td>Full</td>
<td>78</td>
<td>11.74 (5.24)</td>
<td>16.10 (7.88)</td>
</tr>
<tr>
<td>No Inequality</td>
<td>No</td>
<td>54</td>
<td>12.41 (4.53)</td>
<td>13.28 (6.32)</td>
</tr>
<tr>
<td>Medium Inequality</td>
<td>No</td>
<td>72</td>
<td>11.82 (5.17)</td>
<td>16.43 (7.77)</td>
</tr>
<tr>
<td>High Inequality</td>
<td>No</td>
<td>72</td>
<td>12.46 (5.04)</td>
<td>16.60 (7.53)</td>
</tr>
</tbody>
</table>
Nalbantian and Schotter (1997) compare a number of group incentive institutions:

- Tournaments;
- Revenue sharing;
- Target-based schemes.

They find that:

- Relative performance schemes more effective than target based schemes;
- Monitoring is effective but very costly.
Alternative contractual solutions: overview

Piece rates appear to be useful tools to boost productivity.

However, their applicability is limited.

While tournaments can be useful alternatives, they lead to high variability in worker behaviour.
Does this mean there is a one-size fits all institution which is perfectly optimal? Dohmen and Falk (2006) study whether subjects sort themselves into different types of contracts based on individual characteristics.

- Tournaments
- Revenue sharing
- Piece rate
Subjects had to perform a series of tasks to determine productivity

- Solving basic algebra tasks.

They were then asked about how hard they had worked and how well they thought they did vis-a-vis others in the experiment.

- Measuring overconfidence
Subjects were then told they needed to work on similar problems for 10 minutes.

They were told they would have to do similar tasks and were asked to choose which contract they preferred.

After the actual task, experimenters collected data on:

- risk attitudes,
- social preferences
- socio-economic characteristics.
Output is higher in variable pay schemes rather than flat wage scheme

▶ Good to know we respond to incentives!

This is due to self-sorting behaviour:

▶ More productive workers choose variable pay;
▶ Less productive workers choose flat pay;
▶ Sorting behaviour driven by overconfidence;
Round pegs in square holes?: Findings

The more risk averse workers are, the more likely it is they prefer flat pay.

Women more likely to pick flat pay schemes, men more likely to pick variable pay
  ▶ Correlated with risk aversion? (Eckel and Grossman, 2003)

People who have fairness considerations less likely to pick a tournament.