Fishery Management: a variation of the standard case

Saint Andrews February 26 2009

Introduction

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4. Value function approach. Viscosity solution of an Hamilton Jacobi equation

Variation of the standard model

$$\begin{aligned} \dot{x}(t) &= f(x(t)) - qx(t)e(t), \quad x(0) = x_0 \\ x(t) &\geq 0, \;\; 0 \leq e(t) \leq E_M \end{aligned}$$

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- $$f(x) = rx^{\gamma} \left(1 - \frac{x}{K}\right), \quad (\gamma > 1)$$

  $$p(x) = \frac{\bar{p}}{1 + \alpha x^{\beta}}, \quad (\alpha > 0, \; \beta > 1)$$

Equivalent problem

$$J(x(.)) = \int_0^{+\infty} e^{-\delta t} \left[\left(p(x(t)) - \frac{c}{qx(t)} \right) (f(x(t)) - \dot{x}(t)) \right] dt$$

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Euler lagrange equation
$C(x) := A'(x) + \delta B(x) = 0 :$ 3 solutions in $(0, K)$.

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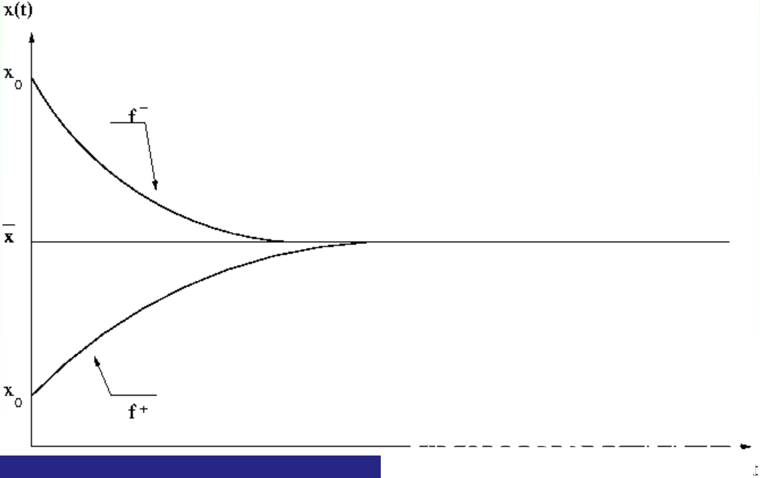

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- As neither $V(.)$ nor $T(.)$ are differentiable. Viscosity solution of HJ.

Results

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Competition at x^* : two optimal solutions exist.