Analysing the Effects of Excise Taxes Using Microsoft Excel

J. Wilson Mixon, Jr. and Bradley N. Hopkins
Campbell School of Business, Berry College

Introduction

Robert L. Bishop (1968) provides a general treatment of specific and ad valorem sales taxes in perfectly competitive industries (those in which firms are price takers) and in simple monopolies (those in which a single seller sells a product for a single price). For competitive industries, Bishop's analysis confirms the standard analysis that appears in textbooks, in which the incidence of a tax imposed on a product is distributed between buyers and sellers according to the shapes of the industry demand curve and the industry supply curve.

Bishop emphatically points out, however, that this analysis does not easily extend to the monopolistic case. Indeed, he calls into question the applicability of the concept of shared incidence in this setting (1968: 215): 'The concept of the "incidence" of a tax is … anomalous…. . In one sense, … the monopolist … pays the whole tax and more; and the burden on consumers must be added to that.' In addition, Bishop shows that it is quite possible that the price that a monopolist charges can rise by more than the specific tax. (Bishop extends the analysis to *ad valorem* taxes, but the analysis here is limited to specific taxes. The generalisation is straightforward.)

The central statement of Bishop's analysis is this (1968: 201): 'As an antidote to excessive preoccupation with the linear case, it is important to notice that the monopolist's price rises either more or less sharply according as [the] demand [for its product] is concave from above or below.' Thus, second derivatives come into play in the case of monopoly, but not in the case of competition. Despite this warning, issued almost four decades ago, textbooks still routinely represent demand curves with straight lines.

Following Bishop, this paper examines the effects of an excise tax imposed on a monopolist's product. The remainder of the paper is organised as follows. The next section reviews salient aspects of Bishop's development. Then simple but quite general polynomial demand and cost curves are introduced and discussed, as is the Microsoft *Excel* workbook that embeds the functions. Finally, exercises based on selected special cases illustrating the use of the workbook are sketched.[1]

Demand, supply and tax incidence in competitive markets

The most pertinent of Bishop's derivations appear below. The firm is assumed to maximise profits, which consist of revenue (R) less cost (C). The following relationships pertain:

- R = pq, where p is the height of the (inverse) demand curve at each quantity (q)
- RN = p + qpN, where pN is the slope of the inverse demand curve (RN = dR/dq is marginal revenue)
- RO = 2pN + qpO, where pO is the rate at which the slope of the demand curve changes as q changes and RO is the rate at which marginal revenue changes as q changes
- C = cq, where c is the height of the average cost curve at each quantity (q)
- CN = c + cpN, where cN is the slope of the average cost curve, dc/dq (CN is marginal cost)
- CO = 2cN + qcO, where cO is the rate at which the slope of the average cost curve changes as q changes and CO is the rate at which marginal cost changes as q changes.

Bishop directly demonstrates the result that every principles textbook reports for a competitive industry:

- dp/dt = pN/(pN – sN), = –pN/(sN +(–pN))

where sN is the slope of the competitive supply curve and dp/dt is the price change per one-unit change in the tax rate. From this result one can quickly determine that some commonly-reported results hold:

- dp/dt = t/2 if the elasticity of demand equals that of supply at the equilibrium price in the absence of taxes (Bishop's analysis is in terms of slope, but p and q are the same for both demand and supply so equal slopes imply equal elasticities and *vice versa*)
- if dp/dt = t/2, then the tax (t) must be equally shared between buyers and sellers in this case
- if –pN > sN (the absolute value of the inverse demand curve's slope exceeds that of the supply curve), dp/dt > ½ (that is, if the demand curve is less elastic than the supply curve, buyers pay more than ½ of the tax)
- if –pN < sN (the absolute value of the inverse demand curve's slope is less than that of the supply curve), dp/dt < ½ (that is, if the demand curve is more elastic than the supply curve, buyers pay more than ½ of the tax),
- dp/dt = 0 if the demand curve is horizontal (all of the tax is absorbed by sellers), and
- dp/dt = 1 if the supply curve is horizontal (all of the tax is paid by the buyers).

Demand, cost and tax incidence in monopoly markets

Bishop's treatment of the competitive market reveals no surprises. Results typically shown in textbooks are demonstrated in a concise fashion. The main point of Bishop's analysis, and the focus of all that follows here, is that the results are much more problematic when the seller faces a downward-sloping demand for its product. In these cases, the curvature of both the demand curve and the average cost curve can affect the way that a tax increase is shared between buyers and the seller.

Now the effect of the tax on price is as follows:

- dp/dt = pN/(RO – CO)

which can be shown to be positive because pN is negative and RN (marginal revenue) must cut CN (marginal cost) from above (so RO < CO at the quantity for which RN = CN. (Otherwise, RN would come to exceed CN as quantity increases, so the firm cannot be at its profit-maximizing quantity.)

Bishop shows that for the special case of linear demand and cost curves, a result very much like that of the competitive case occurs:

- dp/dt = pN/(2(pN – cN)).

Here pN < 0, and cN < 0, so dp/dt is indeed positive. Furthermore, dp/dt is less than 1.0. If cN = 0, then dp/dt = ½. If cN > 0 (a more likely case), dp/dt < ½. The reason is that, in this case the reduction in quantity causes the level of the firm's marginal cost to fall (movement along the MC curve). If the average cost curve slopes downward over the relevant range, dp/dt can exceed 1.0 (the price can rise by more than the excise tax). This happens when the absolute value of the average cost curve's slope is between one-half the demand curve's slope and the value of its slope. Such a steep slope for the average cost curve seems unlikely, but cannot be ruled out theoretically.

The main point of Bishop's development is that dp/dt can exceed 1.0 even if the marginal cost curve is not downward sloping, and that outcome is a function of the curvature of the demand curve. Bishop says:

> 'As an antidote to an excessive preoccupation with the linear case, … notice that the monopolist rises either more or less sharply according as [the demand curve] is concave from above or below. In general, … the effect depends on the slope of the AR [demand curve] relative to the difference in the slopes of the MR and MC. … This the fundamental difference between the monopolistic and competitive cases: the effect under competition depends solely on the first derivatives [slopes] of the demand and supply functions, but under monopoly it depends not only on the first derivatives of AR and MC but also on the second derivative of AR [pO]. Even with constant MC, it is … possible for a specific tax to increase the monopolist's price by more than the tax. This will be so whenever AR is more sharply downward sloping than MR… . In other words, dp/dt is greater than unity when the demand curve's upward concavity is strong enough… (1968: 200).

The model

Exploring the configurations suggested by Bishop requires a specific model of demand and cost. The model outlined below is sufficiently flexible to address the questions at hand. The general forms for the demand and cost functions are these:

$$p = \alpha + \beta q^{-\rho} - \gamma q^{\mu} \quad (1)$$

and

$$c = \lambda + \delta q^{-\varepsilon} + \kappa q^{\nu} \quad (2)$$

Equation (1) is the inverse demand curve, showing willingness to pay. Equation (2) describes the average cost curve. These functional forms are general enough to generate curves with varying curvature characteristics. They include the following important special cases:

- linear demand curve:[2] $\rho = 0$ and $\mu = 1$ ($p = \alpha + \gamma q$)
- constant-price-elasticity demand curve: $\alpha = g = 0$; in this case, the elasticity of demand is $1/\rho$ ($p = \beta q^{-\rho}$)
- constant unit cost (horizontal marginal cost = average cost): $\delta = \kappa = 0$ ($c = \lambda$).

The model is put to work in an *Excel* workbook. The workbook shows the equilibrium value in the absence of a tax. It also shows, in a separate sheet, the optimal value and calculates the deadweight loss that results from the monopolists' production of a quantity below the one for which CN = p. This second sheet becomes important in considering the losses due to the imposition of a tax. The results of imposing a tax on a monopolist are shown in a

á =	
Profit-maximizing q, q* ˜	3.67680823
At q˜ q*, (MR+MC)/2 =	$902.97
At q = q*, p =	$3,861.17
At q = q*, c =	$1,294.87
At q = q*, R' =	$902.97
At q = q*, C' =	$902.97
At q = q*, p' =	-804.557681
At q = q*, c' =	-106.59
At q = q*, p'' =	365.88
At q = q*, c'' =	72.25
At q = q*, R'' =	-263.862167
At q = q*, C'' =	52.46077712
dq/dt =	-0.00316133
dp/dt =	2.543469252

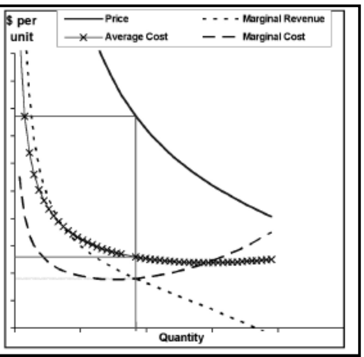

Figure 1. Illustrative example

third sheet. Each sheet requires the discovery of a value that optimises some function. In the first sheet, the optimising condition is that RN = CN, the profit-maximizing rule for the monopolist in the absence of a tax. In the second sheet the optimising condition is that p = CN, the condition for efficient resource allocation. Finally, in the third sheet the optimising condition is that RN = CN + t, where t is the per-unit tax rate.

Equilibrium, no tax

Figure 1 shows the results of implementing Solver given one set of parameter values. We selected these values intentionally to provide a 'reasonable-looking' demand curve (downward-sloping and without obvious curvature anomalies) that, nonetheless, generates a price rise in response to a specific tax that exceeds the tax rate. Along with the graph, Figure 1 shows the critical values for evaluating the effect of an excise tax on the price. Of immediate interest is the last entry, dp/dt = 2.543469252. This implies that a tax of $1 will result in a price increase of about $2.54.

Excel's Solver is required to find the quantity that minimises the absolute difference between marginal revenue (MR) and marginal cost (MC). The table reports the average value of MR and MC as well as each separately. In fact, the two are so close that the difference (0.0000001865) does not appear in the tabled values of MR and MC (RN and CN). To see why the problem must be solved numerically rather than analytically, consider the determination of q*, the profit-maximizing quantity. The marginal revenue and marginal cost functions are as follows:

$$MR = \alpha + (1-\rho)\beta q^{-\rho} - (\mu + 1)\gamma q^{\mu} \quad (3)$$

and

$$MC = \lambda + (1-\varepsilon)\delta q^{-\varepsilon} + (\nu + 1)\kappa q^{\nu} \quad (4)$$

One cannot simply set MR = MC and solve for q because the polynomial resulting from the equality may be of any order.

Efficiency

As does Bishop, this paper focuses mainly on price but also addresses the question of efficiency. Accordingly, once the demand and cost curves are specified and the equilibrium values are determined, the user is invited to examine a second sheet to see the implications of monopoly for efficiency. In this case, the efficient quantity is 8.24 units rather than the equilibrium value of 3.68 units. The deadweight loss associated with monopoly is $6,497.72. To get a sense of how large the deadweight loss is, consider that the equilibrium expenditure on this good is about $3861.67*3.6768 or $14,198.59. That is, the deadweight loss is almost one-half as large as spending. We emphasise that the demand curve is not necessarily representative.

Equilibrium, with tax

We now turn to the effect of imposing an excise tax on this good. The output from the first sheet predicts that the price will rise by about $2.54 per $1.00 tax. In fact, the rise is a bit larger, $3.065 per $1.00 when a $200 per-unit tax is imposed. Also of note is the size of the added deadweight loss (DWL) when the tax is imposed. The addition to DWL is $2,109.81, over three times the amount of tax revenue raised, $605.89. Thus it costs the private sector $605.89 + $2,109.81 or $2,715.70 to deliver $605.89 to the government. This is a striking example of what Bishop means when he says (1968: 105):

> 'The concept of the "incidence" of the tax as between the consumers and the monopolistic producer is even more anomalous than in the competitive case, because of the intensified deadweight loss. In one case, … the monopolist … pays the whole tax and more; and the burden on consumers must then be added to that. With so much deadweight loss, there does not seem to be any meaningful way of saying what fractions of the tax are paid by consumers and producers.'

Exercises

Once the model has been reviewed, the workbook allows the user to specify values and examine their implications for the impact of a specific tax. We provide values for five cases:

- Linear demand curve. The user is invited to confirm the textbook findings regarding the sharing of the tax between seller and buyers. Two exercises are provided. The first retains the U-shaped average cost curve shown in Figure 1. In the second, the case of AC = MC = constant is examined. The user confirms that the monopoly quantity is one-half the efficient quantity and that the price rises by one-half as much as the tax.

- Constant-elasticity demand curve. The user sees that in this case, if the marginal cost curve is horizontal the firm engages in a type of mark-up pricing and, as shown by Mixon (1986) the price rises by more than the tax.

- Upward-sloping MR curve. Economists offer both theoretical and empirical reasons to expect that virtually all demand curves are downward sloping. Textbook representations of demand curves and marginal revenue curves typically show the latter below the former and downward sloping as well. A downward-sloping demand curve need not, however, imply a downward-sloping marginal revenue curve. Formby, Layson and

Smith (1982) show that many of the demand curves that authors of classic textbooks have drawn to illustrate the demand/marginal revenue relationship, in fact imply marginal revenue curves that have some upward-sloping region (though the authors mistakenly drew their corresponding marginal revenue curves as downward sloping).

- Downward-sloping MC curve. Bishop (1968: 101) points out that a tax can raise price by more than the tax rate if the marginal cost curve is sufficiently downward sloping. Such might be the case with a 'natural monopolist'.
- Concave demand curve. This case in included mainly for the sake of completeness. Like the linear case, the concave demand curve yields the result that $dp/dt < 1$.

Summary

This paper develops the analysis of the effect of an excise tax on the price charged by a profit-maximising monopolist. Following Bishop, it shows that the impact of an excise tax imposed on a monopolist differs from that of a competitive industry in two important ways. First and more striking is the result that the price can rise by more than the tax and that this can happen under conditions that cannot be ruled out *a priori*. The second difference is the size of the deadweight loss imposed on a monopolist: this loss can be a multiple of the amount of tax revenue raised. These points are driven home by demonstrating their occurrence in a set of downloadable exercises based on a simple model and executed in a Microsoft *Excel* workbook, which is also available for downloading.

Notes

1 An overview that contains a link to the workbook along with a set of exercises is available at: http://csob.berry.edu/faculty/economics/taxincidence/taxincidence.html.

2 In this familiar case, $RO = 2pN$ (the MR curve has twice the slope of the demand curve). Bishop (1968: 200) shows that $dp/dt = pN/(RO - CO)$, so that $dp/dt > 1$ only if $-pN > -CO$. That is, the price rises by more than the tax only if the marginal cost curve is negatively sloped and it is more steeply sloped than the demand curve. But the latter result violates the second-order condition: the firm would be minimising profits and not maximising them.

References

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Contact details

J. Wilson Mixon, Jr. (Corresponding author)
Dana Professor of Economics
Campbell School of Business
Berry College
E-mail: wmixon@berry.edu

Bradley N. Hopkins
Economics Major
Berry College