# Threshold Concepts in Quantitative Finance

**by Richard Diamond & Holly Smith**

Centre for the Advancement of Learning and Teaching, University College London

1-19 Torrington Place, London WC1E 6BT richard.diamond@ucl.ac.uk, holly.smith@ucl.ac.uk

## What is the threshold concept?

Two definitions, imaginative and practical, point towards opportunity and integrative potential that an understanding of a threshold concept offers, as follows:

1. A threshold concept is akin to a portal, opening up a new and previously inaccessible way of thinking about something. It represents a transformed way of understanding, or interpreting, or viewing something without which the learner cannot progress (Meyer and Land 2005).
2. The threshold concept approach is concerned with how students can be helped to acquire integrating ideas (Davies and Mangan 2007b:18).

A description of the threshold concept's qualities and well-maintained collection of resources can be found on The Threshold Concept Portal at www.ee.ucl.ac.uk/~mflanaga/thresholds.html

## Threshold concept research and practice

The agenda for disciplinary pedagogic studies done within the threshold concept paradigm (TCP) was summarised by Cousin (2006) as follows:

1. Recognising central concepts in contemporary 'stuffed' curriculum.
2. Finding the better ways of linking thinking and practicing. The paradigm suggests to re-work basic understandings that were prematurely closed and train to achieve fluency in operating with procedural knowledge (see a list of procedural knowledge for quantitative finance).
3. Dealing with troublesomeness of knowledge acquisition. It stems from the opening up of concepts deemed understood, necessity to master (a volume of) procedural knowledge, and uncertainty about outcomes of these learning exercises (e.g., whether it will be possible to operate using the new knowledge and use it rather than memory to achieve desired grades).

There is pedagogical value in identification of certain disciplinary concepts as thresholds. It should increase the instructor's awareness about difficulties learners experience as well as realistic level of their progress. For quantitative disciplines, it means to anticipate temporary regress in quality of understanding and reinforce knowledge with practice. Time spent on such re-working brings more benefit than pushing for an acquisition of new isolated knowledge (Davies and Mangan 2007a:725).

Knowledge of disciplinary threshold concepts allows targeted design of exercises and, overall, more efficient, easy and successful application of teaching and learning effort. It should also help to avoid top-down imposition of expert knowledge that will be taken 'as is', memorised, reproduced in assessment, because this learning dynamics will block deeper understanding and independent thought that comes from motivation to match knowledge from a variety of sources.

## Quantitative Finance

Quantitative finance is a discipline of high conceptual density. It builds on every kind of model ranging from physical diffusion processes to probabilistic change of measure. While pedagogic studies of threshold concepts were done for economics, maths (calculus) and accounting, nothing comparable was published for finance. A one-line summary of threshold concept is as follows:

**Ito's lemma, change of measure, risk neutrality, incomplete markets, value vs. price**

## Basic, discipline-specific, and procedural threshold concepts

| Type of Change | Type of Transformation | Examples in Quantitative Finance |
|---|---|---|
| Basic | Concepts that have relevance to everyday experience.<br><br>Basic concepts are re-worked using disciplinary concepts (special expert understanding is formed) | Short selling. Derivative (vanilla, exotic, replication).<br>Discrete vs. Continuos time<br>Spot vs. forward rate (leading to HJM model)<br>Volatility (and its modelling)<br><br>Risk-neutrality. Delta-hedging<br>No arbitrage (or arbitrage condition).<br><br>Portfolio Management (as paradigm linking to economics)<br>Insurance vs. derivatives trading (e.g., speculation) |
| Disciplinary | Acquisition of a perspective that allows to see fundamental similarities among variety of models for different asset classes.<br><br>Integration of personal experience with ideas from discipline. | Stochastic Calculus. Ito's lemma application<br>Risk-neutral and forward measures<br>Fundamental Asset Pricing Formula<br><br>Market price of risk. Unhedged or tail risk<br>Complete vs. incomplete markets<br>Value vs. price<br><br>Likelihood. Stationarity. Correlation vs. co-integration |

| Type of Change | Type of Transformation | Examples in Quantitative Finance |
|---|---|---|
| Procedural | Understanding and mastery of disciplinary modelling procedures enables discipline-specific arguments and ways of practising. | Visual memory. Ability to visualise and sketch<br>Ability to recognise the nature of an equation and parse its structure (e.g., variables, parameters, operators)<br>Taylor series expansion. Ito's lemma<br>Differential equations and solution methods<br>Structural equation models (a system of equations) |
| | Mathematical ability transformation through 'the magic' of working through a proof (by hand) | "Lets construct a portfolio" technique (Replication)<br>Pricing equations and their solutions (Black-Scholes)<br>Constant-to-stochastic parameterisation<br>Jump diffusion<br>Probability and stochastic processes: Geometric Brownian Motion, Poisson Process (Intensity Models)<br>Expectations algebra. Conditioned probability<br>Change of measure (Girsanov Theorem, numéraire)<br>Finite Difference schemes<br>Monte Carlo (uniformity in multiple dimensions)<br>Financial time series modelling methods: PCA, VAR, GARCH, VECM. Special procedures, such as CADF |

In order to facilitate retention of threshold concepts, a pocket-size textbook, Frequently Asked Questions in Quantitative Finance, was developed aiming "to get across the important concepts" (Wilmott 2009).

## Bibliography

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