Threshold Concepts in Quantitative Finance

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In This Presentation...

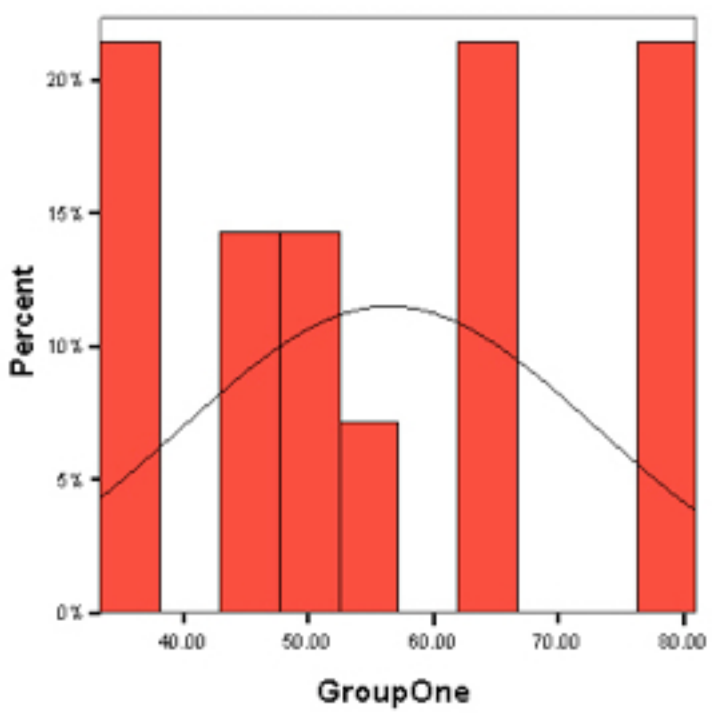
- Challenges of quantitative instruction. Evidence for surface-to-deep learning dynamics
- Introduction to **the threshold concept** as a paradigm
- Quantitative Finance: the volume of procedural knowledge and **‘a maths sweet spot’**
- Illustrations and ideas for knowledge integration (special learning situations) in quantitative finance

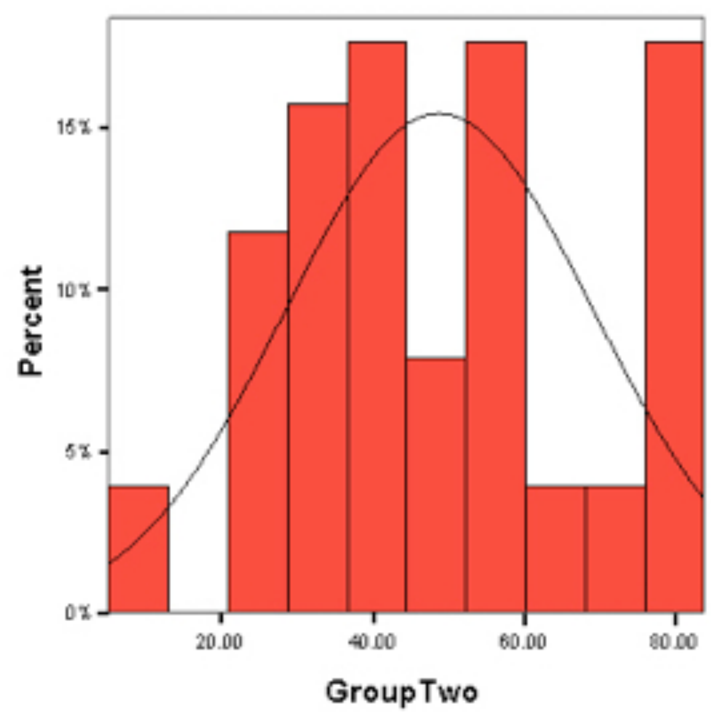
Quantitative Instruction: The Problem(s)

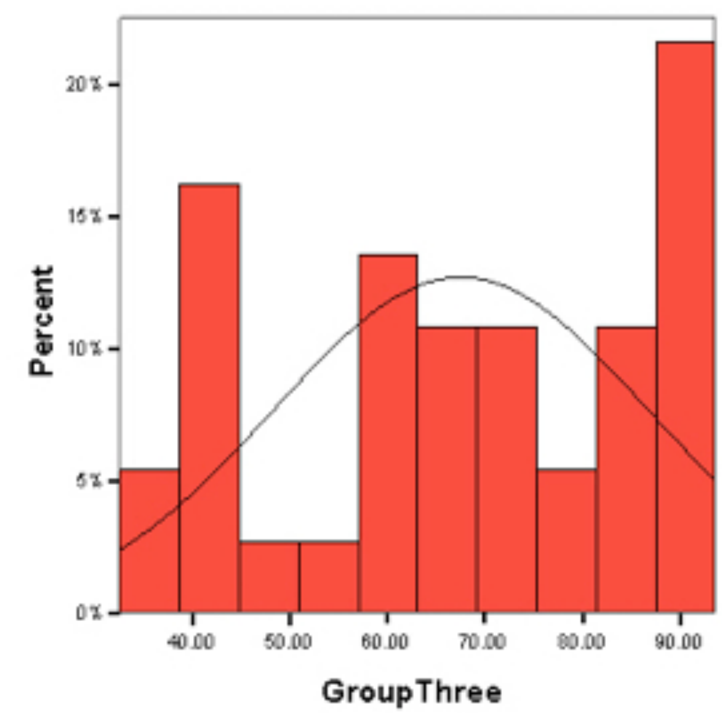
- Students acquire formal knowledge but seem unable to use it when making sense of experience
- Students struggle with underpinning theory and resort to verbatim learning of isolated aspects of a subject, being unable to use them in conjunction
- Students are unable to transfer their skills outside specific, structured problems

If struggling with gaps in what students actually understood please consult *A Handbook on Threshold Concepts in Economics: Implications for teaching, learning and assessment*

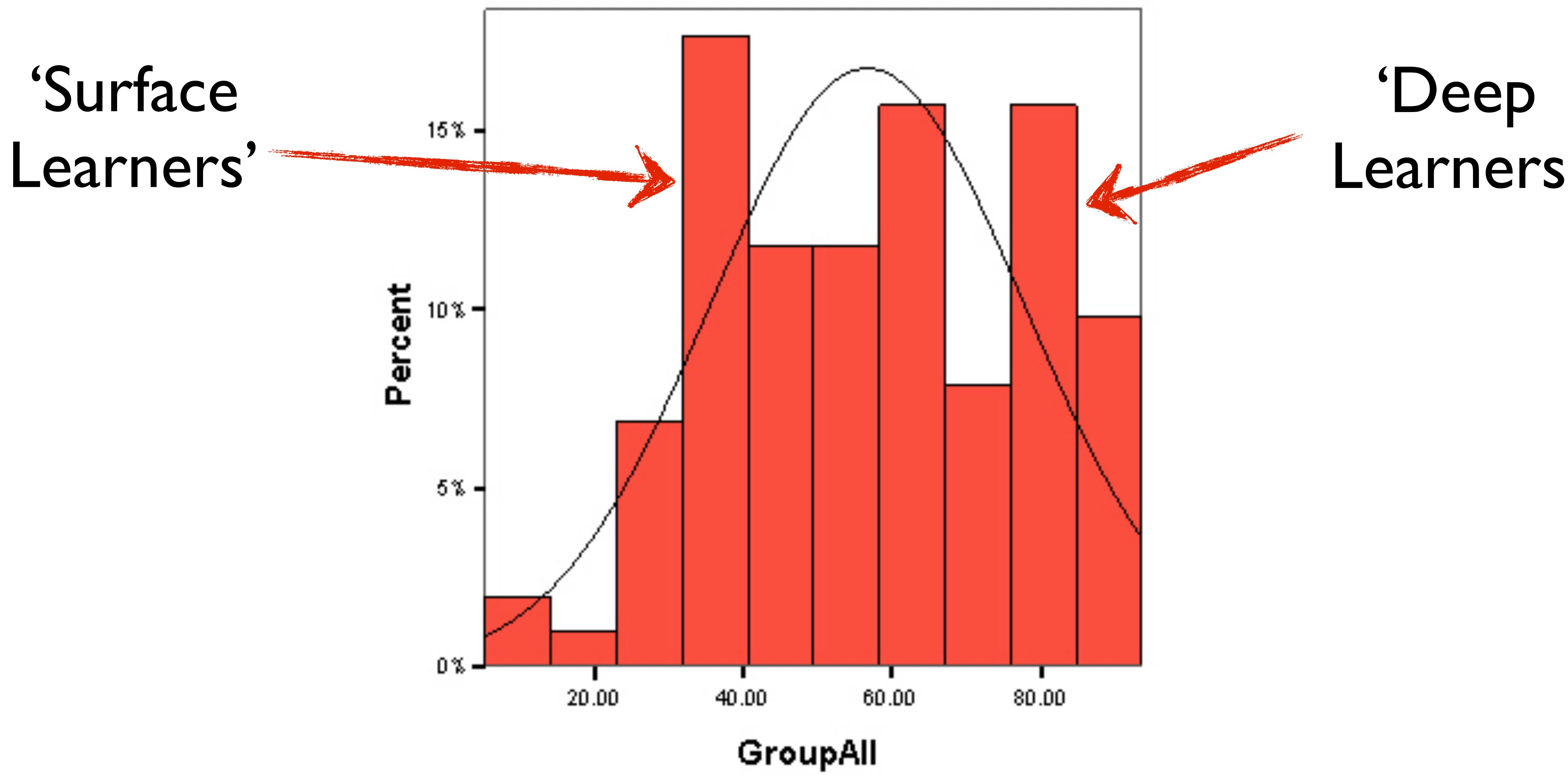
Grade Distributions from UG Statistics Modules







Where is the central tendency?



Multi-Modal Distribution

Reflects a distribution of learning styles *chosen* by students. Only several start with an in-depth style

Definitions

"A threshold concept is akin to a portal, opening up a new and previously inaccessible way of thinking about something. It represents a transformed way of understanding, or interpreting, or viewing something without which the learner cannot progress." (Meyer & Land 2005)

"The threshold concept approach is concerned with how students can be helped to acquire integrating ideas." (Davies & Mangan 2007b)

"The threshold concept approach helps to anticipate challenges of quantitative teaching." (Diamond & Smith 2011)

A well-maintained collection of resources can be found on
The Threshold Concept Portal
at www.ee.ucl.ac.uk/~mflanaga/thresholds.html

‘One Line’ Examples

- Maths: complex number, a limit, the Fourier transform
- Economics: opportunity cost, price elasticity
- **Quantitative Finance: Ito's lemma, change of measure, risk neutrality, incomplete markets**

A couple of questions:

- Which concepts did *your students* experience the most difficulty with?
- Were you satisfied with progress of your students in understanding of those concepts and disciplinary modelling (e.g., econometrics)?

A Big Box of Tools for Quantitative Finance

- The discipline utilises techniques of pure and applied mathematics (analysis & measure theory; stochastic calculus & PDE solution methods) and statistics

- There are discipline-specific modelling techniques: replication of instrument with a portfolio and pricing via expectations (FATF & Feynman-Kac)

Option pricing formulae were known explcitly from 1960 but real significance was in how they were derived by B-S. Delta-hedging and independence of expected rate of return were the great discoveries awarded the Nobel Prize (Haug 2007: 39-40).

Finance maths is simple: PDEs are parabolic and numerical methods are well-specified

Example: Black'76 Formula for Fixed Income Derivatives

The proof of Black's (1976) formula involved the following mathematics: differential equations, Brownian Motion, stochastic calculus and Ito's integral (Ito's lemma), double change of measure (Girsanov theorem) to obtain a futures martingale measure, Feynman-Kac theorem, and assumptions allowing to use the Normal Distribution.

The following techniques specific to quantitative finance were also utilised: forward price (given discounting in continuos time), self-financing trading strategy, no arbitrage valuation, FAPF and Black-Scholes solutions formulae.

That was an extremely brief overview of a dense 8-page proof

A Maths Sweet Spot

- The idea is advocated by Paul Wilmott (2009)

  “The models should not be too elementary so as to make it impossible to invent new structured products, but nor should they be so abstract as to be easily misunderstood by all except their inventor (and sometimes even by him), with the obvious and financially dangerous consequences.”

- Example of CDO and CDO^2 products that show ‘bizarre’ correlation behaviours (to default) that are different for each tranche, leading to ‘off the cliff’ scenario for value

- Tendency to make *presentation* of finance research overly complex (unlike experts, students cannot discern a paper)

Outside of the sweet spot, *model risk* is increased

‘Live’ Integration

- The following Illustrations show how expert practitioners, who were faculty and tutors on CQF programme, mapped concepts during their sessions
- Programme participants reported ‘a transformative impact’ on their knowledge, generated by experiences of dynamic mapping of disciplinary concepts

Understanding of the Illustrations requires familiarity with the type of modelling done in quantitative finance

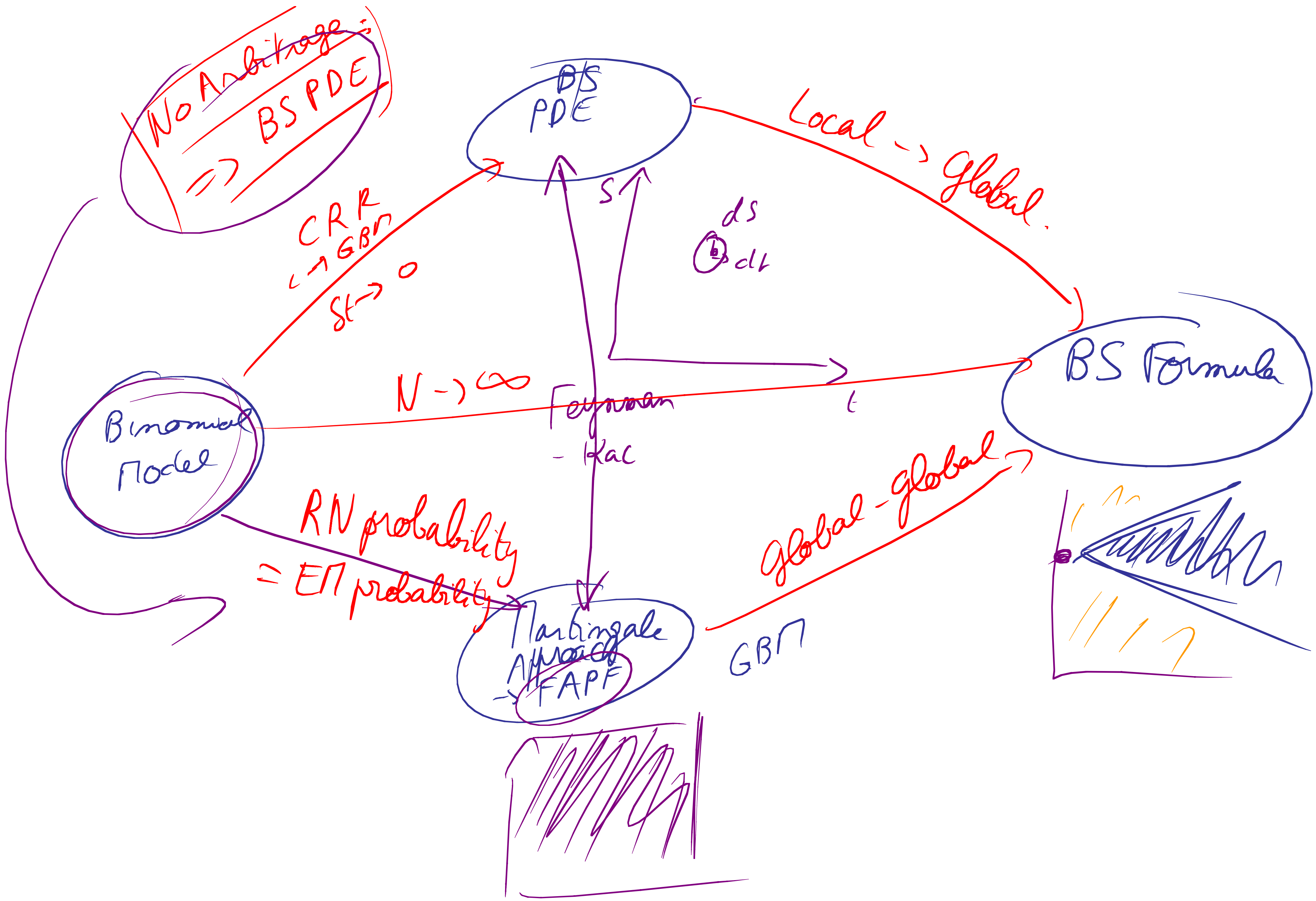


Illustration 1 compares Black-Scholes pricing equation that applies locally (pricing over the next small period) to FAPF that applies globally (over the life of a product)

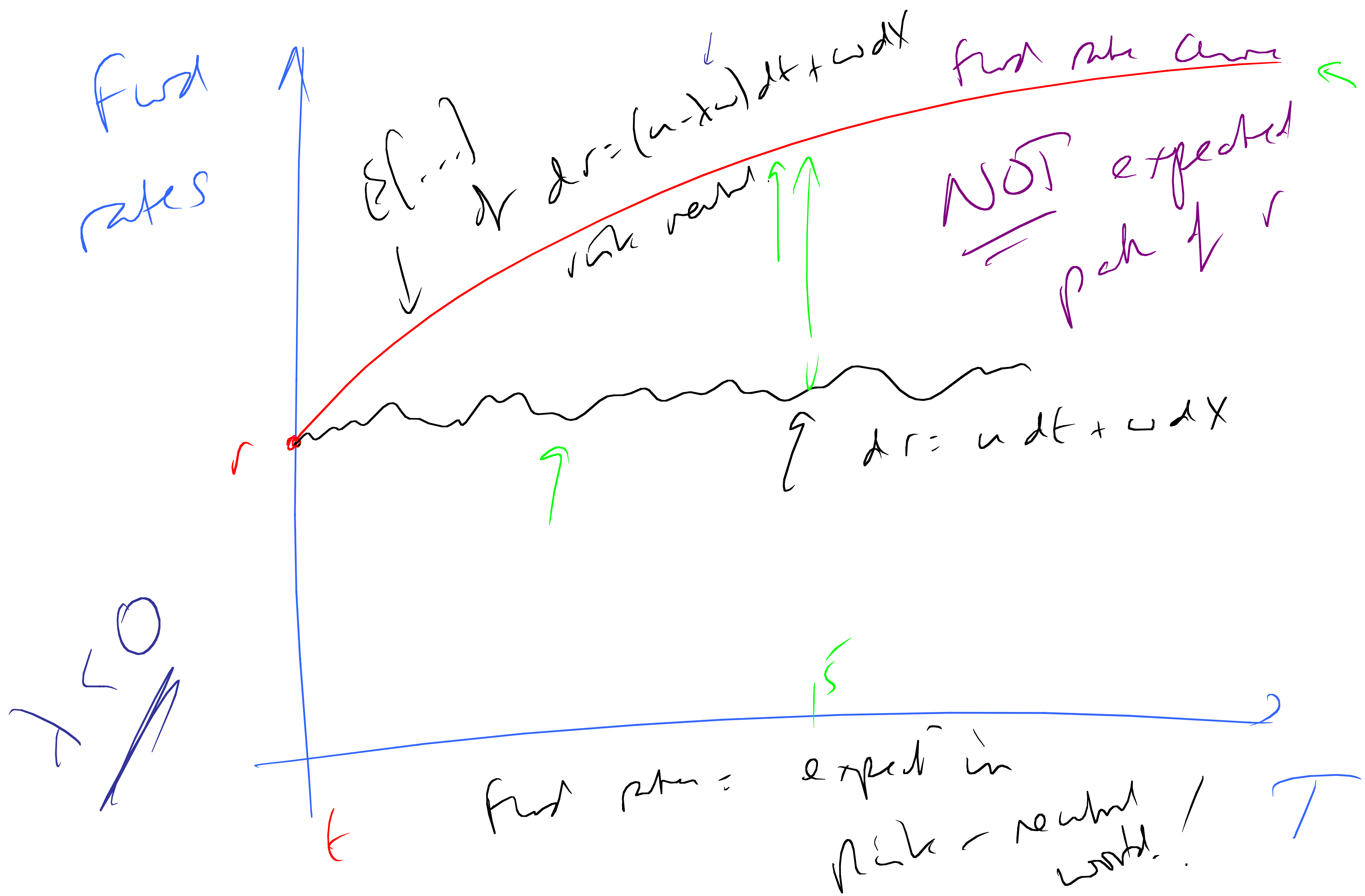


Illustration 2 aims to show a critical difference between risk-free and forward rates: forward rate curve is **not** an expected path of a mean-reverting risk-free rate

Popular one-factor spot-rate models

The real spot rate r satisfies the stochastic differential equation

$$dr = u(r,t)dt + w(r,t)dX. \tag{4}$$

Model	$u(r,t) - \lambda(r,t)w(r,t)$	$w(r,t)$
Vasicek	$a - br$	c
CIR	$a - br$	$cr^{1/2}$
Ho & Lee	$a(t)$	c
Hull & White I	$a(t) - b(t)r$	$c(t)$
Hull & White II	$a(t) - b(t)r$	$c(t)r^{1/2}$
General affine	$a(t) - b(t)r$	$(c(t)r - d(t))^{1/2}$

Here $\lambda(r,t)$ denotes the market price of risk. The function $u - \lambda w$ is the risk-adjusted drift.

For all of these models the zero-coupon bond value is of the form $Z(r,t;T) = e^{A(t,T) - rB(t,T)}$.

Illustration 3 relates the parameters (e.g, drift and process volatility) of several named stochastic interest rate models to the same parameters in one generalised model

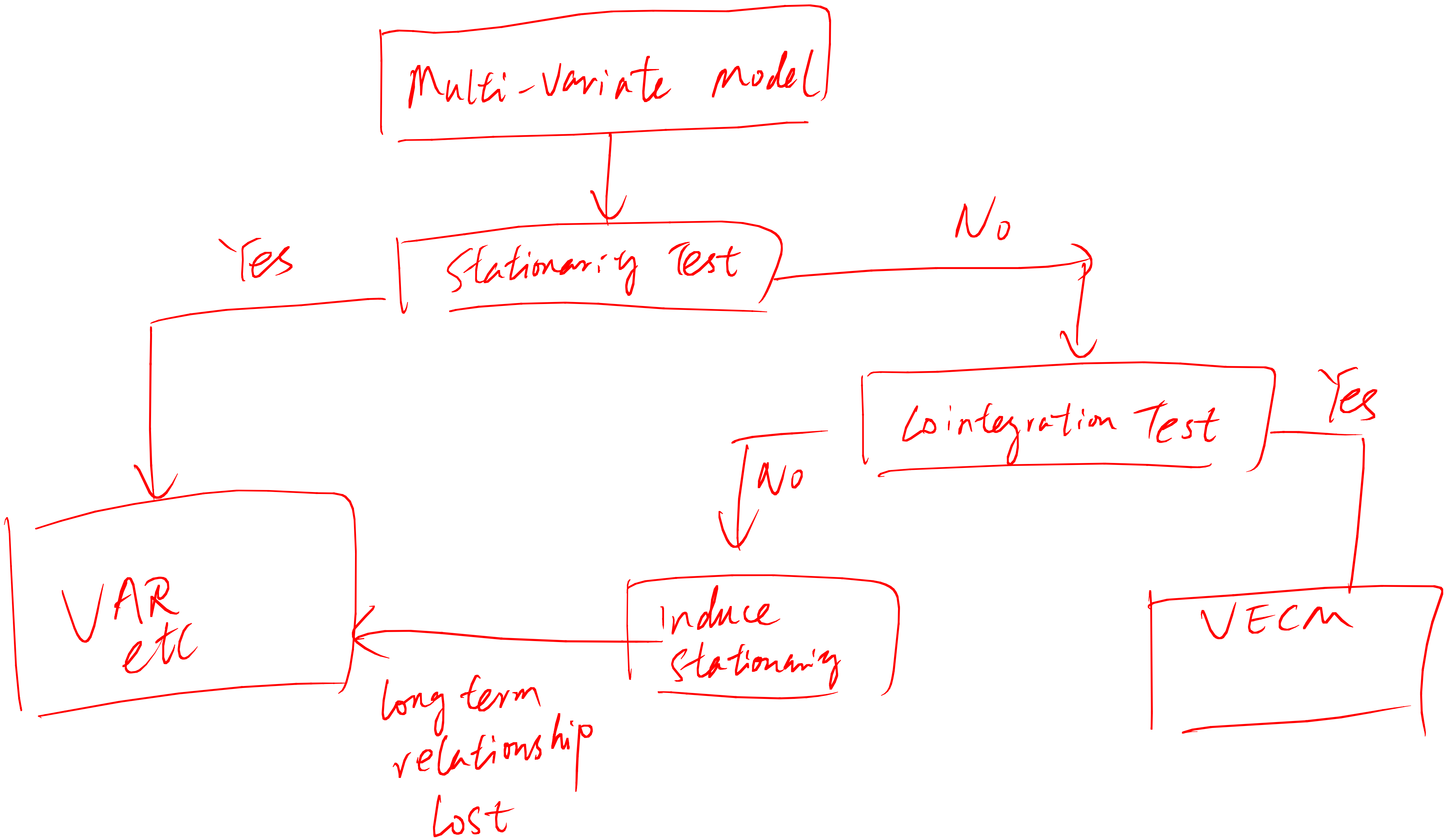


Illustration 4 is a flow chart that utilises ‘black boxes’ to present sophisticated statistical procedures for testing and modelling of long-term relationships in a simple form

Learning Situations Design

- Go through mathematical proofs ‘by hand’ and make learners comfortable with volumes of it
- Teach initial modelling skills without the formal disciplinary apparatus
- Structure situations in which insight comes from the re-working of prior knowledge and replacing of simple ways of understanding

For an example, see DEE 2007 Keynote on Teaching Undergraduate Econometrics by Prof. David Hendry
http://www.economicsnetwork.ac.uk/dee2007/

Assessment in Quantitative Subjects

- Provide scaffolding for knowledge integration *early in the module*: hand out past exam papers at the first lecture and suggest portable textbooks
- Design assessment so as to remove incentives for surface learning: include questions on reasoning, model derivation and output interpretation
- Be realistic about what is possible. In cases such as a six-week Masters module, we can only bring specific mathematical and modelling skills up to a standard
- If things are tight, choose a project over exam

This was the first glance at *pedagogic value* of the threshold concept approach.

With feedback and suggestions for further enquiry, please get in touch via

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