Liquidity Trap in an Inflation-targeting Framework: A Graphical Analysis

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Abstract
This paper presents a simple New Keynesian model with alternative assumptions regarding the conduct of monetary policy. The central bank is assumed to either follow a Taylor rule or minimise a social welfare loss function. The model can be tractably described by means of a straightforward graphical apparatus, which, so far, has not been extended to include the treatment of the liquidity trap. The paper presents an analysis of the zero nominal interest rate bound using this apparatus and discusses the implications of pre-emptive monetary easing when the macroeconomic conditions suggest that the bound may restrict future monetary policy effectiveness.

JEL classification: A22, E32, E52

1. Introduction

On 16 December 2008, the Federal Open Markets Committee established ‘a target range for the federal funds rate of 0 to 1/4 percent’. For several weeks prior to this announcement, the effective federal funds rate and the yield on Treasury bills had hovered in that range, making the FOMC announcement a mere recognition of existing reality and raising the practical relevance of the zero bound on the nominal interest rate for the conduct of monetary policy. The United States has thus joined Japan whose own experience with the liquidity trap in the late 1990s provided the impetus for the recent efforts to study monetary policy in this environment.

The recent theoretical literature on this subject extends the now standard New Keynesian models (see Clarida, Gali and Gertler (1999) and Woodford (2003) for a comprehensive treatment) into the setting where the nominal interest rate is bound by zero. This strand of literature suggests that monetary policy should be more accommodative towards inflation than in the models where this bound does not feature. Eggertson and Woodford (2003) study the effect of this constraint in the context of a forward-looking model with perfect foresight and show that the effects of the liquidity trap can be reduced by the central bank’s credible commitment to keep the nominal interest rates low even after the effects of a negative demand shock have passed. This allows forward-looking agents to act on the promise of an inflationary environment with negative real interest rates in the present and hence lifts the economy.

1 The author thanks the editor and an anonymous referee for valuable suggestions. All remaining errors are the author’s.

2 Svensson (2003a) and Jeanne and Svensson (2007) discuss mechanisms for escaping the liquidity trap in the context of an open economy, such as Japan. This paper focuses on the treatment of issues that arise in the closed-economy setting.
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out of the liquidity trap sooner. Adam and Billi (2006, 2007) study the conduct of optimal monetary policy under commitment and discretion without perfect foresight, in a stochastic environment, and find that the zero nominal interest rate bound makes it optimal for the central bank to pursue the reduction of nominal interest rates more aggressively and pre-emptively in its response to negative demand shocks than in the case where liquidity trap is not a possibility. Kato and Nishiyama (2005) analytically derive the optimal monetary policy reaction function when the zero nominal interest constraint binds in the context of a backward-looking deterministic model and find that it is more aggressive and expansionary than the rule that is optimal in the absence of that bound.

The main contribution of the present paper is to develop the graphical apparatus for discussing these results that should be accessible to an audience of undergraduate students and non-specialists. Starting with the work of Romer (2000) and Taylor (2000), modern monetary theory has been rendered more accessible to wider audiences. Carlin and Soskice (2005) survey several alternatives and propose a tractable model with backward-looking dynamics. Bofinger, Mayer and Wollmershäuser (2006) build on their work to present extensive graphical analysis of the conduct of monetary policy in the inflation-targeting framework. Guest (2003) and Turner (2006) present alternative specifications of inflation-targeting models that facilitate their exposition in the classroom. Furthermore, several undergraduate textbooks, such as Carlin and Soskice (2006) and Jones (2007), have used a New Keynesian model with adaptive inflationary expectations as the framework for analysing short-run fluctuations. Weise (2007) shows that a pedagogical version of the New Keynesian model can be extended to incorporate additional considerations, such as the term structure of interest rates. Kapinos (2010) provides a description of the several versions of the model using an Excel workbook. This paper considers a setup that is similar to the ones studied in this body of work and extends it to examine the effect of the zero nominal interest rate bound.

The present paper also studies the consequences of uncertainty surrounding the possibility that a large negative demand shock – the standard source of the liquidity trap – may (or may not) materialise in the future. A central bank’s ability to anticipate a large negative demand shock and hence engage in pre-emptive monetary easing that is shown to be optimal in the context of this paper’s model may dramatically reduce social welfare losses relative to the scenario where no pre-emptive action is undertaken. However, pre-emptive easing will generate social welfare losses if the shock fails to materialise. This paper provides a framework for studying the tradeoffs involved in this setup.

The rest of the paper is organised as follows. The following section describes the changes in the standard analytical apparatus that result from the zero nominal interest rate bound for two versions of a model of short-run dynamics under alternative assumptions regarding the conduct of monetary policy. We then argue in favour of a pre-emptive monetary easing when the future macroeconomic conditions make the zero nominal interest rate bound a distinct possibility using the graphical apparatus developed in the previous section. Finally, the article concludes.

2. Monetary policy rules in a New Keynesian model with adaptive inflationary expectations

This section presents a brief overview of the now standard three-equation model that has been used for the analysis of monetary policy. First, the household sector Euler equation describes the negative relationship between a measure of real activity, such as the output gap, and the real interest rate. Alternatively, it can be motivated by the standard Keynesian treatment of the aggregate expenditure function. Second, the firms’ first-order condition with respect to their own prices in a staggered pricing framework gives rise to inflation as a positive function of output gap. Finally, the monetary authority is assumed to follow either an instrument rule where it sets the nominal interest rate in response to deviations of inflation from the long-term target and output gap or a targeting rule where it minimises a
social welfare loss function period by period. The former was proposed by Taylor (1993) as a descriptor of the conduct of monetary policy in the United States and the latter can be derived as a second-order approximation of the household utility function.

**Instrument version**

A popular way to model a central bank's decision to set the nominal interest rate is by means of a Taylor rule, whereby the target rate is a function of inflation, output gap and possibly other variables. Clarida, Gali and Gertler (2000) provide a comprehensive study that analyses the Taylor rule in the context of the US data. This monetary rule (referred to as the MR schedule in this paper) has the pedagogical advantage of discussing the systematic conduct of monetary policy without resorting to calculus. The instrument version of the model, therefore, is defined by the standard IS, aggregate supply (AS), and MR equations:

\[
x_t = \sigma r^* - \sigma (i_t - \pi_t) + \epsilon_t^x, \tag{1}
\]

\[
\pi_t = \pi_{t-1} + \kappa x_t + \epsilon_t^x, \tag{2}
\]

and

\[
i_t = i^* + \gamma (\pi_t - \pi^*) + \gamma_x x_t + \epsilon_t^x, \tag{3}
\]

where \(\pi^*\) is the inflation rate targeted by the central bank, \(r^*\) is the long-run real interest rate that is consistent with the output gap of zero and, as it follows, \(i^* = r^* + \pi^*\); \(\pi_t\) is inflation, \(x_t\) is the output gap and \(i_t\) is the nominal interest rate. The so-called Taylor principle states that the stabilising monetary policy requires that the central bank should respond to a percentage increase in inflation by raising the nominal interest rate by more than 1%, hence \(\gamma \pi > 1\). Empirical studies of versions of the model that feature forward-looking expectations, such as Cho and Moreno (2006) and Rabanal and Rubio-Ramirez (2005), suggest that the values for the elasticity of intertemporal substitution, \(\sigma\), slope of the Phillips curve, \(\kappa\), and the degree of central bank’s responsiveness to output gap, \(\gamma\), all fall in the range between 0 and 1. Plugging (3) into (1) and solving for inflation, one can obtain the aggregate demand (AD) relation:

\[
\pi_t = \pi^* + \frac{\epsilon_t^x - \sigma \epsilon_t^i}{\sigma (\gamma_n - 1)} - \frac{\sigma \gamma_x + 1}{\sigma (\gamma_n - 1)} x_t. \tag{4}
\]

Together with (2) it determines the equilibrium output gap and inflation; the latter is taken as exogenous in (1) and (3). Note that the Taylor principle ensures that the slope of aggregate demand is negative.

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4 Svensson (2002, 2003b) introduced this ‘instrument’/‘targeting’ nomenclature to distinguish between a monetary policy response function that sets the nominal interest rate as a linear function of inflation and output gap and the central bank’s minimisation of a social welfare loss objective.

4 Note that since the IS/MR diagrams below will have the nominal interest rate on the vertical axis, the graphical version of the IS schedule can be written as:

\[
i_t = r^* + \pi_t - \frac{1}{\sigma} x_t + \frac{1}{\sigma} \epsilon_t^x,
\]

Unlike the models where the real interest rate features in the IS/MR plane, both IS and MR will shift due to changes in inflation. Introducing this minor complication, however, provides a more direct way to study the effect of the zero bound on the nominal interest rate.
As in the Keynesian tradition, the zero nominal interest rate bound on monetary policy becomes effective due to a large negative demand shock, \( \varepsilon^*_{t} < 0 \).\(^1\) Formally, the condition for the requisite size of the negative demand shock to generate the liquidity trap is given by the following inequality:

\[
epsilon^*_{t} \leq -\frac{\sigma \kappa (\nu_{t+1} - 1) + \sigma \nu_{t} + 1}{\kappa \nu_{t} + \nu_{t}^{*}} \nu_{t}^{*} + \frac{\nu_{t} - 1}{\kappa \nu_{t} + \nu_{t}^{*}} \pi^{*} + \frac{\sigma \nu_{t} + \nu_{t}}{\kappa \nu_{t} + \nu_{t}^{*}} (\pi_{t-1} + \varepsilon^*_{t}) - \frac{1 - \sigma \kappa}{\kappa \nu_{t} + \nu_{t}^{*}} \varepsilon^*_{t}.
\]

This inequality has an intuitive interpretation. To make the bound effective, the negative demand shock has to overcome the positive pressure on the interest rate coming from a contractionary monetary policy shock,\(^6\) the long-run real interest, a cost-push shock and the long-run inflation target. Note that, since in the long run \( \pi_{t-1} = \pi^{*} \), the coefficient on the long-run inflation target will be\(^6\)

\[
- \frac{1 - \sigma \kappa}{\kappa \nu_{t} + \nu_{t}^{*}} < 0.
\]

This has important implications to be discussed more fully below. A higher choice for the long-run inflation target by the central bank makes it less likely that a negative demand shock of a given size will generate the liquidity trap by providing an inflationary cushion.\(^7\)

Once the negative demand shock is sufficiently large, the IS and MR equations change to reflect the zero nominal interest rate bound. The latter is now simply given by \( i_{t} = 0 \), while the IS schedule (1) and, correspondingly, AD schedule (4) become, respectively:

\[
x_{t} = \sigma r^{*} + \sigma \pi_{t} + \varepsilon^*_{t},
\]

\[
i_{t} = r^{*} + \pi_{t} - \frac{1}{\sigma} x_{t} + \varepsilon^*_{t}.
\]

This suggests that the IS schedule becomes vertical and the AD schedule becomes upward sloping. The latter result is obtained because the central bank will no longer be able to engage in stabilising monetary policy and lower the nominal interest rate more than proportionately in response to a given decrease in inflation, because the nominal interest rate is at zero. Furthermore, comparing the slopes of

\(^{1}\) In particular, solving (2) and (4) for equilibrium inflation and output and using (3) to determine the equilibrium interest rate, we have:

\[
i_{t} = r^{*} - \frac{\nu_{t} - 1}{\sigma \kappa (\nu_{t+1} - 1) + \sigma \nu_{t} + 1} \pi^{*} + \frac{\sigma \nu_{t} + \nu_{t}}{\sigma \kappa (\nu_{t+1} - 1) + \sigma \nu_{t} + 1} (\pi_{t-1} + \varepsilon^*_{t})
\]

\[
+ \frac{1 - \sigma \kappa}{\sigma \kappa (\nu_{t+1} - 1) + \sigma \nu_{t} + 1} \varepsilon^*_{t}.
\]

Setting this equation equal to zero and solving for the demand shock, one can obtain the size of the shock necessary to make the zero nominal interest rate constraint binding. If the liquidity trap is induced by a negative supply shock, the economy will first experience lower inflation and a positive output gap, as the AS schedule shifts down, and then, as adaptive expectations start working, the AS will keep shifting down along the upward-sloping portion of the AD schedule until output gap is zero. Hence the economy will settle at a lower inflation rate and zero output gap, a scenario not nearly as dramatic as the model’s response to large negative demand shock.

\(^{6}\) Clearly, the central bank would generate large costs in terms of a negative output gap by engaging in a discretionary contraction while the demand shock is large and negative.

\(^{7}\) There are additional benefits associated with higher inflation targets. Sensitivity analysis with respect to key parameters of the infinite horizon model laid out in the Appendix and impulse response functions of the variables to an anticipated future negative demand shock are available from the author upon request. These results show that when the shock materialises, the relative gains from pre-emptive easing discussed below decline sharply, suggesting that guessing whether or not a sizeable negative shock will realise will be less necessary. See Billi (2009) for a derivation of a positive optimal inflation rate under model uncertainty and occasionally bounding zero nominal interest rate and Billi and Kahn (2008) for a tractable discussion of issues driving this result.
(7) and (2), a straightforward determinacy condition must be satisfied for unique equilibrium inflation and output gap to exist:

$$\frac{1}{\sigma} > \kappa.$$  \hspace{1cm} (8)

This is, the upward-sloping branch of AD must be steeper than AS. If this condition does not hold, a unique equilibrium will not exist in this model.

**Figure 1** Transitory vs. permanent negative demand shock in the instrument model

Figure 1 demonstrates the effect of a large negative demand shock using the graphical representation of the model's schedules, assuming that the model is characterised by the long-run equilibrium at time $t=0$. For simplicity, the figure assumes that the shock at $t=1$ is such that the equilibrium nominal interest rate exactly equals 0. The left panel assumes that the shock is transitory and fully disappears at $t=2$. The IS schedule shifts down from $IS_0$ to $IS_1$ due to the shock and the effect of lower equilibrium inflation. The MR shift from $MR_0$ to $MR_1$ is also motivated by lower equilibrium inflation, but, once the zero bound on the nominal interest rate is reached, it becomes horizontal. Since the central bank is no longer able to follow the Taylor principle and respond to falling inflation by decreasing the nominal interest rate by even larger amounts, the AD schedule becomes upward-sloping. At $t=2$, firms peg their inflationary expectations to the equilibrium level of $\pi$ at $t=1$ and the AS schedule shifts down to $AS_2$. However, by then the effect of the transitory shock on the AD schedule has disappeared, hence AD returns to $AD_2=AD_0$. This means that $\pi_2 > \pi_1$ and, as inflationary expectations adjust adaptively, the economy eventually returns to the long-run equilibrium.
The right panel of Figure 1 demonstrates that for the liquidity trap to be a salient issue in this model, the negative demand shock has to be sufficiently persistent. In this panel, the demand shock is permanent: $\text{AD}_0$ shifts down to $\text{AD}_1=\text{AD}_2$ and stays in that position. Now that lower equilibrium inflation pushes the AS schedule downwards in subsequent time periods along the upward-sloping demand schedule, the model does not have a stable equilibrium: output gap and inflation will both fall indefinitely.\(^8\) As inflation falls, the vertical IS schedule shifts farther to the left.

Avoiding the liquidity trap, therefore, is necessary to prevent economic destabilisation in this setup. The next section will discuss how this may be done by the central bank’s engaging in pre-emptive expansionary monetary policy. Alternatively, instructors may discuss the use of fiscal policy that here is modelled by the demand shock, $\varepsilon_t$. A negative demand shock that comes, for instance, from a dramatic decrease in asset prices may be offset by means of a fiscal expansion. Timing issues are also critical here: the longer the fiscal policy makers wait to implement a stimulus package, the deeper the economy will sink into a recession.

Figure 1 also illustrates the recent ostensibly puzzling finding of Eggertsson (2008): an adverse cost-push shock, say, due to higher unionisation rates that raise firms’ labour costs, will have beneficial effects on output gap and inflation and help the economy escape from the liquidity trap. This result has an intuitive interpretation. The main problem in a liquidity trap is falling inflation, which, since the nominal interest rate is at zero, raises the real interest rate and depresses the output gap that, in turn, lowers inflation even further. A cost-push shock raises inflation and hence breaks this vicious cycle. Of course, the benefits of these higher labour costs pertain only to the situation when the zero nominal interest rate bound is effective. Implementing policies that permanently increase firms’ costs may be hard to reverse, once the economy recovers from the liquidity trap.

### Targeting version

Svensson (2002, 2003b) has long criticised the use of Taylor rules as descriptors of monetary policy. Although they do seem to summarise a large proportion of a modern central bank’s decision-making process, no central bank has formally committed to following such a rule. Instead, the conduct of monetary policy may be more adequately described by the central bank’s choosing a nominal interest rate so as to minimise a discounted stream of social welfare losses given by the weighted average of squared departures of inflation from its target level and output gap from its target level of zero. Carlin and Soskice (2005, 2006) provide a graphical description of this problem in the case where the zero nominal interest rate does not place a bound on the conduct of monetary policy.\(^9\)

In this sub-section, instead of following an instrument rule, such as (3), the central bank conducts optimal monetary policy by minimising a social welfare loss objective on a period-by-period basis.

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\(^{8}\) Note that this does not happen with respect to a permanent negative demand shock that is small enough not to induce the liquidity trap. In that case, the economy will eventually converge to zero output gap and lower long-run level of inflation; this point is discussed in the following section. Furthermore, it should be clear that the sustained fall in output gap and inflation is due to adaptive inflationary expectations. A similar, if smaller in magnitude, result would hold if a fraction of firms anchored their expectations to the central bank’s inflation target and the remaining firms continued to form their expectations adaptively. It is also possible to study the effect of a permanent demand shock with forward-looking expectations, as in Eggertson and Woodford (2003); however, that setup does not lend itself easily to graphical analysis.

\(^{9}\) Wiese (2007) also moves away from the Taylor rule description of monetary policy and allows the central bank to set the interest rate exogenously. Postulating a social welfare loss function provides a criterion for the level that the central bank may want to choose.
without taking into consideration the effect of adaptive inflationary expectations on future outcomes.\(^10\)

Algebraically, the central bank solves:

$$
\min_{\pi_t, x_t} \left\{ \pi_t - \pi_t^* \right\}^2 + \alpha x_t^2 \right\}
$$

subject to (2). The first-order condition to this problem is the aggregate demand schedule:

$$
\pi_t = \pi_t^* - \frac{\alpha}{k} x_t.
$$

and the interest rate consistent with implementing this optimal policy can be recovered from (1).

Ordinarily, it is only a cost-push shock that represents a trade-off between stabilising inflation and output gap and generates a social welfare loss; relatively small demand shocks can be offset by adjusting the nominal interest rate, such that \( \Delta \pi = \frac{1}{\sigma} \pi_t^* \). However, if the negative demand shock is sufficiently large, the zero nominal interest rate bound becomes effective and the central bank will not be able to offset the shock fully.\(^11\) The zero nominal interest rate bound, therefore, will become binding if:

$$
\epsilon_t \leq -\sigma \pi_t^* + \frac{k(1-\sigma k)}{\alpha k^2 + \alpha} \pi_t^* - \frac{\alpha \sigma + k}{\alpha(k^2 + \alpha)} (\pi_t + \pi_t^*),
$$

where the intuitive interpretation is similar to (11). If this conditions holds, then the central bank will not be able to act in the optimal manner, hence aggregate demand will not be given by (10) but by (7). That is, the AD schedule will again become upward-sloping, if condition (11) is met and inflation is decreasing.

Figure 2 illustrates how the targeting model responds to a permanent negative demand shock at time \( t=1 \) starting in the long-run equilibrium at \( t=0 \). The left panel shows the effect of a shock that is sufficiently small to be offset without the nominal interest rate falling to zero. The shock pushes the IS schedule to the left\(^12\) but the central bank can pick a point on this schedule that fully offsets the effect of the shock on the position of the AD schedule.

\(^{10}\) The setup where the central bank minimises the expected discounted stream of social welfare losses is dealt with algebraically in the next section.

\(^{11}\) Solving for the equilibrium output gap and inflation using (2) and (10) and plugging the result into (1), we can find the equilibrium interest rate as:

$$
\epsilon_t = r - \frac{k(1-\sigma k)}{\alpha k^2 + \alpha} \pi_t - \frac{\alpha \sigma + k}{\alpha(k^2 + \alpha)} (\pi_t + \pi_t^*) + \frac{1}{\sigma} \epsilon_t^*.
$$

One can then set the nominal interest rate to zero and solve for the demand shock to obtain its value that would trigger the liquidity trap.

\(^{12}\) The graph shows the final position of the IS schedule that also incorporates the change in equilibrium inflation.
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Figure 2 Liquidity trap in the targeting model: small vs. large shock

The right panel of Figure 2 demonstrates what happens when picking the optimal point on the new IS schedule would imply an equilibrium nominal interest rate that is less than zero. Now that the central bank is bound by the zero interest rate constraint, it will operate on the upward-sloping portion of the AD schedule given by (7), as it is unable to achieve a bliss point with respect to the social welfare loss function. The ellipse centred on the long-run equilibrium point describes the social welfare loss that is thus incurred, assuming that $0 < \alpha < 1$, i.e. that the central bank cares more about stabilising inflation than output gap. As in the right panel of Figure 1, once the equilibrium outcome happens on the upward-sloping portion of the AD schedule, the AS schedule will start shifting down, increasing the magnitude of the negative output gap, lowering inflation, and generating ever larger social welfare losses.

13 For $\alpha = 1$, the social welfare loss will be represented by a circle; for $\alpha > 1$, it will be represented by an ellipse elongated along the vertical axis.
3. **A case for pre-emptive monetary easing**

In early 2008, the US Federal Reserve pursued an aggressive series of interest rate cuts that was unprecedented in its scope. At an emergency meeting on 22 January, the Federal Open Market Committee (FOMC) made the first 0.75% cut in the target federal funds rate (FFR), the largest single-day decrease since the early 1980s. This action came a week before the regularly scheduled meeting on 30 January, when the FOMC cut the FFR target by another 0.5%. On 18 March, the FOMC decided to further the cut by another 0.75% and followed up on 30 April, reducing the target rate by another 0.25% to 2%. The last two actions generated some disagreement within the FOMC, with Richard Fisher (President of the Dallas Fed) and Charles Plosser (President of the Philadelphia Fed) voting against the measures both times. The FOMC press releases acknowledged inflationary risks and the decisions drew heavy criticism from a number of prominent monetary economists. For instance, Rogoff (2008) described the United States as ‘ground zero for global inflation’. The series of interest rate cuts undertaken by the Fed then took a pause until the fall of 2008.

Why did the Fed undertake this bold policy action? Although, given the extent of the financial crisis that unfolded in the fall of 2008 with short-term interest rates hitting zero in December, it may seem that the Fed was insufficiently aggressive in its cuts, in real time, its decision-making seemed extraordinarily expansionary. With some measures of inflation, such as the Consumer Price Index, registering annualised increases in excess of 4%, the highest level in about two decades, the Fed’s focus on stimulating the real side of the economy was far from uncontroversial. This section attempts to provide a graphical apparatus that should facilitate the explanation of possible alternatives to an advanced undergraduate audience.

The Appendix lays out the theoretical motivation for pre-emptive monetary easing. Instead of myopically following the optimisation problem described by (9), the central bank optimises the discounted stream of social welfare losses over the infinite horizon, recognising the effect of adaptive inflationary expectations on future outcomes. Although dynamic optimisation may be accessible to some students, the technical treatment of this problem is considerably more complex than the simple framework discussed above. Graphical treatment should be more than sufficient to illustrate how engaging in monetary expansion prior to an anticipated negative demand shock can improve macroeconomic outcomes. As before, the following two sub-sections discuss this scenario first in the instrument version of the model and then in the targeting one.

*Pre-emptive easing in the instrument model*

Suppose that the central bank receives information that, in the next time period, there is a high probability that a large negative – and sufficiently persistent – demand shock will hit the economy. Given the discussion in the previous section, it should be clear that, if any monetary policy action is delayed until the shock hits, the economy will enter into a protracted recession, with no self-correcting mechanism to return it to the long-run equilibrium output gap of zero. If the central bank ordinarily sets the nominal interest rate using a Taylor rule (3), it may want to avoid this situation by pre-emptively applying a negative exogenous monetary shock, \( \varepsilon_0 < 0 \), before \( \varepsilon_1 < 0 \) hits the economy in the subsequent time period. That is, given the prevailing conditions at \( t=0 \), the central bank will lower the nominal interest rate below the level suggested by the systematic component of the Taylor rule.

Figure 3 details this process: in the left panel, \( \varepsilon_1 < 0 \) does not materialise, whereas in the right panel it does. Pre-emptive easing accomplishes a downward shift of the MR schedule, from \( MR_0 ^{NPE} \) – the
benchmark of no pre-emptive easing – to MR₀ PE. ¹⁴ The lower interest rate stimulates aggregate demand that shifts from AD₀ NPE to AD₀ PE. The higher equilibrium inflation will ensure that at t=1 aggregate supply shifts up due to higher adaptive inflationary expectations. Since the shock does not materialise in the left panel, the central bank will have to generate a monetary contraction at t=1, shifting the MR schedule to MR₁ PE. Relative to no pre-emptive action where the economy would be at rest in the long-run equilibrium, this will generate a recession to make inflation return to the target level. Furthermore, note that the central bank can accelerate the process of inflation returning to its long-run level by raising the nominal interest to a level exceeding that given by (10), if it wants to avoid, say, the loss of credibility associated with higher inflation.

The right panel of Figure 3 explains the difference between pre-emptive easing and strictly following (3) if ε₁ < 0 does materialise. In the absence of pre-emptive easing, the economy falls into the liquidity trap and the subsequent dynamics follow as per the discussion that accompanied the right panel of Figure 1. With pre-emptive easing, however, the central bank has generated an inflationary cushion that allows for the nominal interest rate to stay positive, giving the monetary authority room for additional action. For simplicity, the figure assumes that under pre-emptive easing the central bank offsets the demand shock, so that the aggregate demand schedule returns to the long-run position.

¹⁴ The final position of MR₀ PE accounts for the fact the pre-emptive easing gives rise to higher equilibrium inflation at t=0 due to the upward-sloping AS schedule.
Pre-emptive easing in the targeting model

This sub-section builds on the analysis carried out above and considers the possibility of departure from the optimal leaning-against-the-wind rule (10) when the central bank anticipates a large negative demand shock in the subsequent time period. In that case, the central bank may want to choose to lower the nominal interest rate pre-emptively, pushing out aggregate demand and generating a larger inflationary cushion that will make the onset of a liquidity trap in the next time period less likely.

The left panel of Figure 4 shows the effect of this pre-emptive easing, if the negative shock fails to materialise at $t=1$. The central bank lowers the nominal interest rate at $t=0$, which generates inflation above the long-run target level and pushes the IS schedule up to $IS_0^{PE}$. Higher inflation at $t=0$ will trigger higher inflationary expectations at $t=1$, shifting aggregate supply to $AS_1^{PE}$. (As before, in the absence of pre-emptive easing, the AS schedule would have remained at its original position.) Since the shock actually does not materialise, the AD schedule returns back to the original position at $t=1$. The central bank now leans against the inflationary wind and generates a negative output gap to bring inflation down to the level given by its long-run target.

The right panel of Figure 4 describes the effect of pre-emptive easing if the shock does materialise. In the absence of pre-emptive easing, the zero bound on the nominal interest rate becomes effective in $t=1$, which generates a relatively large social welfare loss in that time period and, if the shock is sufficiently persistent, more – and potentially larger – social welfare losses in the subsequent time periods. With pre-emptive easing, however, the central bank has enough leeway to use the nominal interest rate to offset the effect of the negative demand shock, possibly returning the AD schedule to its long-run position, with a much smaller social welfare loss.

Figure 4  Pre-emptive easing in TM: (left) shock doesn’t materialise; (right) shock does materialise
4. Conclusion

This paper has developed the graphical apparatus for studying the effects of the zero nominal interest rate bound in the now standard model of short-run fluctuations with adaptive inflationary expectations. It has formally derived the result in the context of this model that an optimising central bank should engage in pre-emptive easing if the zero bound may be effective in the future. Furthermore, it has related its main findings to the recent policy-making challenges faced by the US Federal Reserve.

Of course, the model studied here is intentionally simplified, so that it can be captured graphically in an accessible fashion. In so doing, it has abstracted from important issues, such as the role of the forward-looking behaviour and the loss of inflation-fighting credibility that engagement in pre-emptive easing may bring. Exploring this agenda is an ongoing effort that will undoubtedly intensify in the near future.

References


Appendix: Theoretical motivation for pre-emptive easing

This appendix derives the theoretical motivation for pre-emptive easing along the lines of Kato and Nishiyama (2005) who also work with a backward-looking model following Ball (1999) and Svensson (1997). The model in this paper has a simpler lag structure, which allows us to arrive at the key result in a very accessible analytical fashion. Since inflationary expectations are backward-looking, this analysis is similar to the finding of Eggertsson and Woodford (2003) who show that with forward-looking inflationary expectations the liquidity trap can be avoided by promising to generate excess inflation, even after the effect of a persistent negative demand shock has passed.

Although minimising (9) provides a straightforward way to derive an aggregate demand relation that can be easily graphed, in practice, monetary authorities are concerned with the effect of their policy on future social welfare outcomes as well. Hence the more general formulation of the central bank’s problem in the context of the model used in above is:

\[
\min_{\pi_t, x_t, i_t} \frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \pi_s - \pi^* \right)^2 + \alpha x_s^2 \]

subject to (2), (1), and the nominal interest non-negativity constraint:

\[
i_t \geq 0.
\]

The Lagrangian associated with this problem is:

\[
L = \frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \pi_s - \pi^* \right)^2 + \alpha x_s^2 - \lambda_t^{AS} \left( \pi_s - \pi^* + \kappa x_s + \epsilon_s^x - \pi_s \right) - \lambda_s^{FS} \left( \sigma - \sigma(is - ns) + \epsilon_s^x - x_s \right) - \lambda_s i_s.
\]

As Kato and Nishiyama (2005) emphasise, it is important to ensure that the Lagrangian multipliers are positive, because they will enter the central bank’s optimal policy rule, signalling the direction of its adjustment relative to the case where (A2) is not binding. The first-order conditions with respect to \( \pi, x \) and \( i \) are:

\[
\begin{align*}
\pi_t - \pi^* + \lambda_t^{AS} - \alpha \lambda_t^{IS} - \beta E_t \lambda_{t+1}^{AS} &= 0, \\
\alpha x_t - \kappa \lambda_t^{AS} + \lambda_t^{IS} &= 0, \\
\sigma \lambda_t^{IS} - \sigma \lambda_t &= 0.
\end{align*}
\]

If (13) does not bind, then \( \lambda_t^s = \lambda_t^{IS} = 0 \) and \( \lambda_t^{AS} = \frac{\alpha}{\kappa} x_t \). Hence for a positive equilibrium interest rate, it is only aggregate supply that may impose a constraint on the conduct of monetary policy.

To see why pre-emptive monetary easing is optimal in this setting, suppose that initially at time period \( t \) the central bank does have room to manoeuvre and (13) does not bind but starting with \( t+1 \) it may. Aggregate demand at time period \( t \) becomes:

\[
\pi_t = \pi^* - \frac{\alpha}{\kappa} x_t + \beta E_t \lambda_{t+1}^{AS},
\]

or,

\[
\pi_t = \pi^* - \frac{\alpha}{\kappa} x_t - E_t \sum_{s=t+1}^{\infty} \beta^{s-t} \left( \pi_s - \pi^* \right) + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} \lambda_s.
\]
Equation (A4) draws an intuitive contrast with (10): the latter is static and easier to graph but does not account for the fact that today’s inflationary outcome will affect future welfare through adaptive inflationary expectations. The other representation, equation (A5), emphasises the role of the zero bound on the nominal interest rate. Insofar as it can become effective in the future and the Lagrangian multipliers associated with it, \( \lambda^i_t \), can be positive, it is optimal for the current inflation rate to be higher than it would have been in the absence of that possibility. This can be achieved through monetary easing in the current time period.

Similar intuition can be gleaned for the instrument model. Although Taylor rules like (3) are imposed on the model exogenously, one could combine (A5) with (1) to back out the optimal interest rate response function:

\[
i_t = r^* + \pi_t + \frac{1}{\sigma} \epsilon_{t} + \frac{\kappa}{\alpha \sigma} (\pi_t - \pi^*) + \frac{1}{\sigma} \epsilon_{t} \sum_{s=t+1}^{\infty} \beta^{s-t} (\pi_s - \pi^*) - \frac{1}{\sigma} \epsilon_{t} \sum_{s=t+1}^{\infty} \beta^{s-t} \lambda^i_s.
\]  

(A6)

This equation has a number of intuitive implications. The Taylor principle is satisfied given (8) and \( 0 < \alpha < 1 \), i.e. the optimal response function will be stabilising under these conditions. Anticipation of future inflation in excess of the target level suggests monetary tightening now, as the central bank recognises that current lower inflation will moderate its future levels through adaptive expectations. Finally, the possibility that the zero nominal interest rate bound may become effective in the future calls for pre-emptive monetary easing now.

**Author Biography**

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