Powers & Exponents

5^3, 2^4, $(-2)^5$, $(-1)^{231}$, 3^1, 3^0, 3^{-1}, 3^{-2}, $4^{\frac{1}{2}}$,

$27^{2/3}$, $3^{-\frac{2}{5}}$, $16^{-\frac{3}{4}}$, $5^3 5^4$, $\left(\frac{3}{2}\right)^4$, $\frac{2^7}{2^3}$, $(3^2)^5$

Calculate the value of $Y = A K^{\frac{1}{4}} L^{3/4}$ C when $A = 30$, $K = 256$, $L = 160\,000$

Solve $x^{2/3} = 4$

Sketch the curves $y = x^2$, $y = x^{\frac{1}{2}}$, $y = 2^x$, $y = 2^{-x}$

Powers & Exponents

$4 \times 3 = 4+4+4 = 12 = 3+3+3+3 = 3 \times 4$

4^3 — exponent / power; 4 — base

$4^3 = 4 \times 4 \times 4$

$= 16 \times 4$

$= 64$

$3^4 = 3 \times 3 \times 3 \times 3$

$= 9 \times 3 \times 3$

$= 27 \times 3$

$= 81$

$t^2 = t \times t$

$x x x x x = x^5$

Powers & Exponents

$5^3 = 5 \times 5 \times 5 = 25 \times 5 = 125$

$2^4 = 2 \times 2 \times 2 \times 2 = 4 \times 2 \times 2 = 8 \times 2 = 16$

$(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -2^5 = -32$

$(-1)^{231} = -1$

$(-1)^2 = (-1)(-1) = 1$

$(-1)^3 = -1$

$(-1)^4 = 1$

$(-1)^5 = -1$

Powers & Exponents

$3^4 =$

$\times 3$ ↑ ↓ $\div 3$

$3^3 = 3 \times 3 \times 3 = 27$

$\times 3$ ↑ ↓ $\div 3$

$3^2 = 3 \times 3 = 9$

$\times 3$ ↑ ↓ $\div 3$

$3^1 = 3$

$\times 3$ ↑ ↓ $\div 3$

$3^0 = 1$

$\times 3$ ↑ ↓ $\div 3$

$3^{-1} = \frac{1}{3}$

$\times 3$ ↑ ↓ $\div 3$

$3^{-2} = \frac{1}{9} = \frac{1}{3^2}$

$0^0 = 1$

$x^0 = 1$

$x^{-1} = \frac{1}{x}$ if $x \neq 0$

$x^{-n} = \frac{1}{x^n}$ if $x \neq 0$

$\frac{1}{x^{-2}} = x^{-(-2)} = x^2$

Powers & Exponents

$5^3 5^4 = 5^3 \times 5^4 = 5\times5\times5\times5\times5\times5\times5 = 5^7 = 5^{3+4}$

$x^a x^b = x^{a+b}$

$x^a y^a = (xy)^a$

$(2\times3)^4 = (2\times3)^4 = 2\times3\times2\times3\times2\times3\times2\times3 = 2\times2\times2\times2\times3\times3\times3\times3 = 2^4 3^4$

$\frac{2^7}{2^3} = \frac{2^7}{2^3} = \frac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2} = 2^4 = 2^{7-3}$

$\frac{x^a}{x^b} = x^{a-b}$

$= 2^7 \frac{1}{2^3} = 2^7 2^{-3} = 2^{7+(-3)} = 2^4$

$\left(\frac{3}{2}\right)^4 = \left(\frac{3}{2}\right)^4 = \frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2} = \frac{3\times3\times3\times3}{2\times2\times2\times2} = \frac{3^4}{2^4}$

$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$

$(3^2)^5 = (3^2)^5 = 3^2\times3^2\times3^2\times3^2\times3^2$

$(x^a)^b = x^{ab}$

$= 3\times3\times3\times3\times3\times3\times3\times3\times3\times3 = 3^{10} = 3^{2\times5}$

Powers & Exponents

$x^1 = x$

$(x^a)^b = x^{ab}$

$4^{\frac{1}{2}}$ Let $y = 4^{\frac{1}{2}}$. So $y^2 = (4^{\frac{1}{2}})^2 = 4^{\frac{1}{2}\times 2} = 4^1 = 4$

$y^2 = 4$

$x^{\frac{1}{2}} = \sqrt{x}$, $x \geq 0$

$y = \sqrt{4} = 2$

$125^{\frac{1}{3}}$

$(125^{\frac{1}{3}})^3 = 125^{\frac{1}{3}\times 3} = 125^1 = 125$

$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

$x^{\frac{1}{n}} = \sqrt[n]{x}$

Powers & Exponents

$(x^a)^b = x^{ab}$

$27^{2/3}$ Let $t = 27^{2/3}$. So, $t^3 = (27^{2/3})^3 = 27^{\frac{2}{3}\times 3} = 27^2$

$t^3 = 27^2$

$27^{\frac{2}{3}} = 27^{\frac{1}{3}\times 2} = (27^{\frac{1}{3}})^2 = 3^2 = \underline{9}$

$81^{-3/4} = \frac{1}{81^{3/4}} = \frac{1}{(81^{1/4})^3} = \frac{1}{3^3} = \underline{\frac{1}{27}}$ $(81^{3/4} = (81^3)^{\frac{1}{4}} = (81^{\frac{1}{4}})^3)$

$3^{-\frac{2}{5}} = \frac{1}{3^{2/5}} = \frac{1}{(3^2)^{1/5}} = \frac{1}{9^{1/5}} = \underline{\frac{1}{\sqrt[5]{9}}}$

Powers & Exponents

Calculate the value of $Y = A K^{\frac{1}{4}} L^{3/4}$ when $A = 30$, $K = 256$, $L = 160\,000$

$K^{1/4} = \sqrt[4]{K} = \sqrt[4]{256} = 4$

$L^{\frac{1}{4}} = \sqrt[4]{160000} = 20$

$256 = 240 + 16$
$= 4 \times 60 + 4 \times 4$
$= 4 \times 64$
$= 4 \times 4 \times 16 = 4 \times 4 \times 4 \times 4 = 4^4$

$160000 = 16 \times 10000$
$= 2^4 \times 10^4 = (2 \times 10)^4$
$= 20^4$

$L^{3/4} = (L^{1/4})^3 = 20^3 = (2 \times 10)^3 = 2^3 \times 10^3$
$= 8 \times 1000$
$= 8000$

$Y = A K^{1/4} L^{3/4}$
$= 30 \times 4 \times 8000 = 3 \times 4 \times 8 \times 10 \times 1000 = \underline{960\,000}$

Powers & Exponents

Solve $x^{2/3} = 4$, $x > 0$

$(x^2)^{\frac{1}{3}} = 4$

$\sqrt[3]{x^2} = 4$

$x^2 = 4^3 = 72$

$x = 8$

$(x^{\frac{1}{3}})^2 = 4$

$x^{\frac{1}{3}} = 2$

$\sqrt[3]{x} = 2$

$x = 2^3 = 8$

Powers & Exponents

Sketch the curves $y = x^2$, $y = x^{\frac{1}{2}}$ $(= \sqrt{x})$, $y = 2^x$, $y = 2^{-x}$ $(= (\frac{1}{2})^x)$