

Linear equations & their graphs

Which of these equations are linear?

$$3x+2=5,$$

$$7 - \sqrt{2}x = 5 + \sqrt{3},$$

$$1-x = 2+3y,$$

$$2x^2 = 3+x,$$

$$\frac{2}{x} = 3,$$

$$(3x+2)(2y+5) = 6$$

Sketch the graphs of $y = 3x - 2$, $y - 3 = 2(x - 5)$, $2x + 3y - 6 = 0$

What is the equation of the straight line through $(2, 5)$ and $(3, 4)$?

Suppose that the graph of $g(x)$ is a straight line and $g(2) = 0$, $g(0) = 6$. What is $g(1)$?

How many solutions (x, y) are there to $\begin{cases} 3x - 2y = 1 \\ 2x + 3y = 5 \end{cases}$?

How many solutions (x, y) are there to $\begin{cases} 3x - 2y = 1 \\ 4y - 6x = -2 \end{cases}$?

equation is linear if it is equivalent to

$$ax = k$$

$$ax + by = k$$

$$ax + by + cz = k$$

⋮

Linear equations & their graphs

Which of these equations are linear?

$$3x + 2 = 5 \Leftrightarrow 3x = 3 \checkmark \Leftrightarrow x = 1$$

$$7 - \sqrt{2}x = 5 + \sqrt{3} \Leftrightarrow \underline{-\sqrt{2}x} = \underline{-2 + \sqrt{3}} \checkmark$$

$$2x^2 = 3 + x \quad \times$$

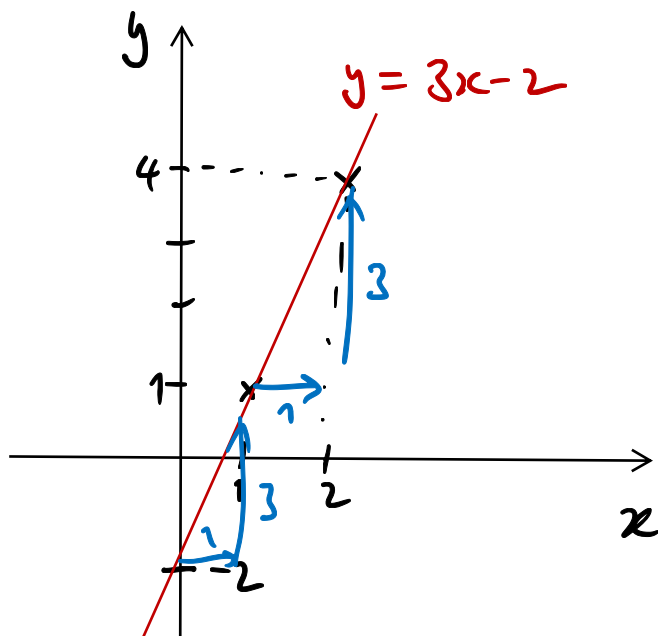
$$\begin{aligned} 1 - x &= 2 + 3y \Leftrightarrow 1 = 2 + 3y + x \\ &\Leftrightarrow -1 = 3y + x \Leftrightarrow x + 3y = -1 \end{aligned}$$

$$\times \frac{2}{x} = 3 \quad \not\Leftrightarrow 2 = 3x$$

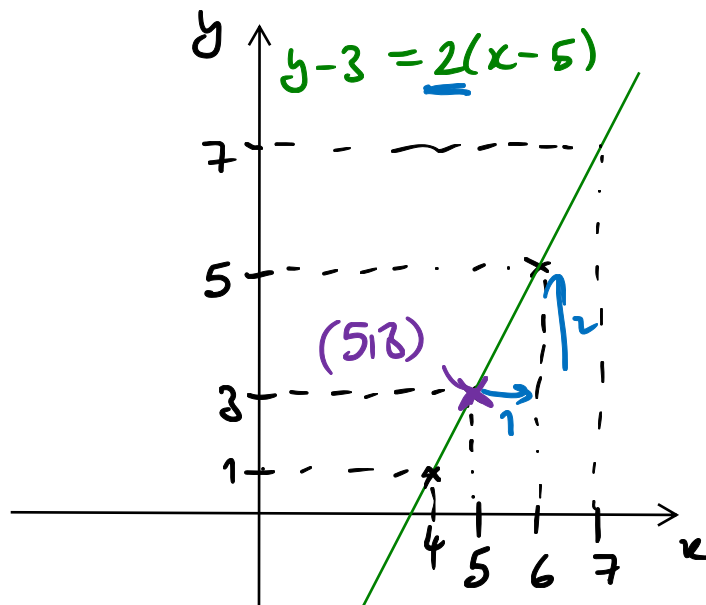
$$(3x + 2)(2y + 5) = 6 \quad \times$$

Linear equations & their graphs

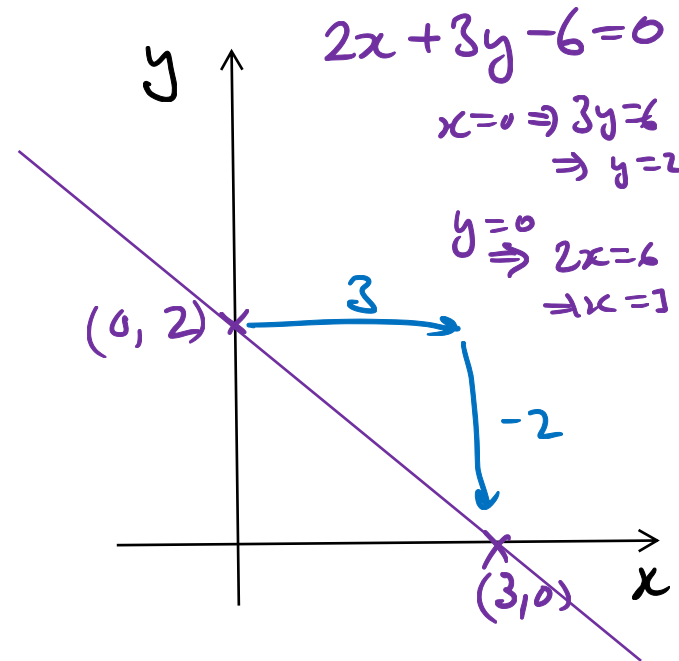
Sketch the graphs of $y = 3x - 2$, $y - 3 = 2(x - 5)$, $2x + 3y - 6 = 0$



$y = mx + c$
 ↑ slope ↑ y intercept



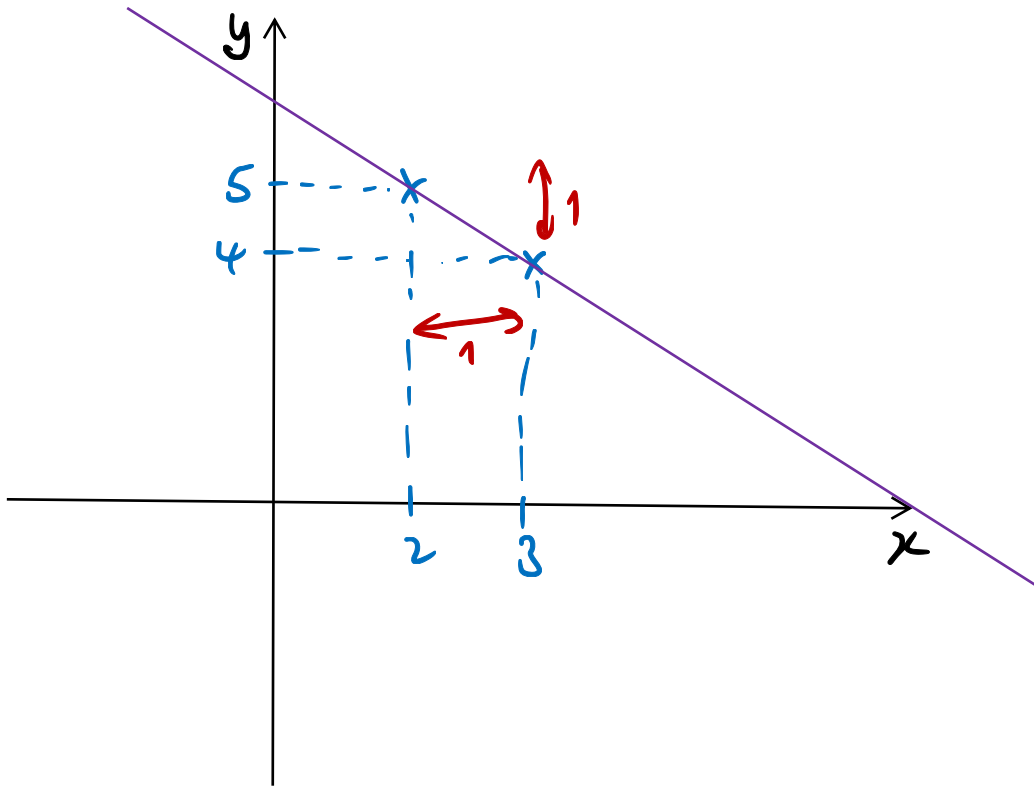
slope
 $y - y_0 = m(x - x_0)$
 go through (x_0, y_0)
 $\frac{y - y_0}{x - x_0} = m$



slope $-\frac{2}{3}$
 $y = -\frac{2}{3}x + 2$

Linear equations & their graphs

What is the equation of the straight line through (2,5) and (3,4) ?



$$y - y_0 = m(x - x_0)$$

$$x_0 = 2$$

$$y_0 = 5$$

$$m = \text{slope} = \frac{-1}{1} = -1$$

$$\underline{\underline{y - 5 = (-1)(x - 2)}}$$

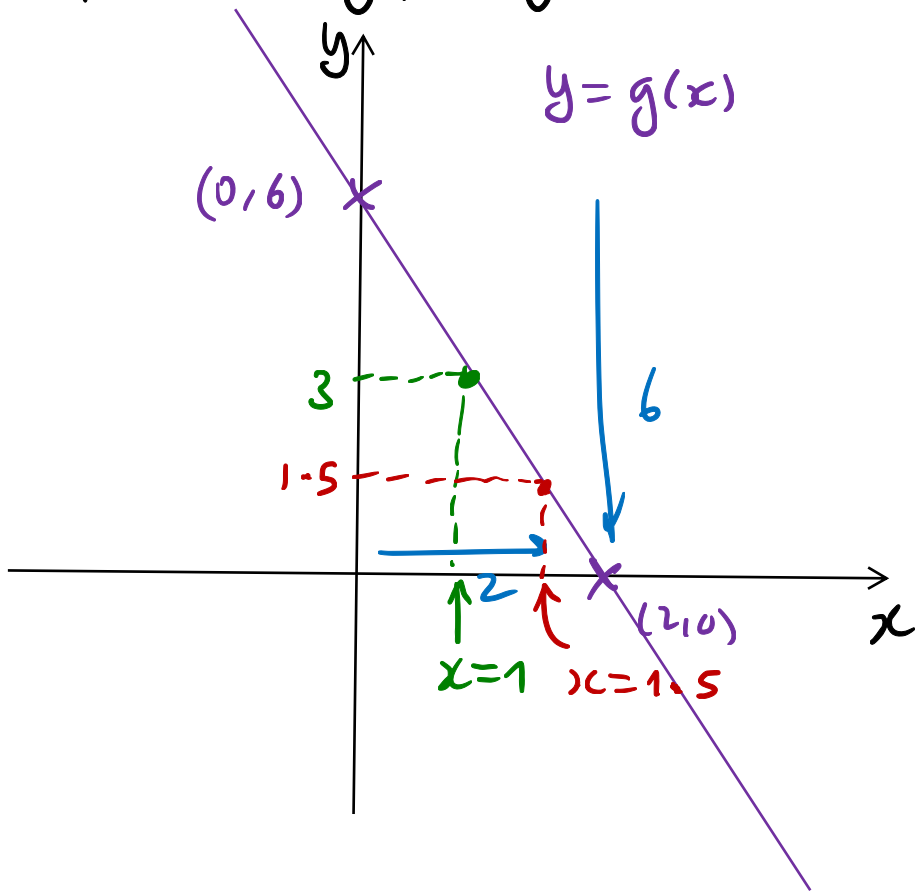
$$y - 5 = -x + 2$$

$$\underline{\underline{y = -x + 7}}$$

$$\underline{\underline{x + y = 7}}$$

Linear equations & their graphs

Suppose that the graph of $g(x)$ is a straight line and $g(2)=0$, $g(0)=6$. What is $g(1)$?



$$y = mx + c$$

$$c = 6$$

$$m = \text{slope} = \frac{-6}{2} = -3$$

$$y = -3x + 6 = g(x)$$

$$g(1) = -3 \times 1 + 6 = -3 + 6 = 3$$

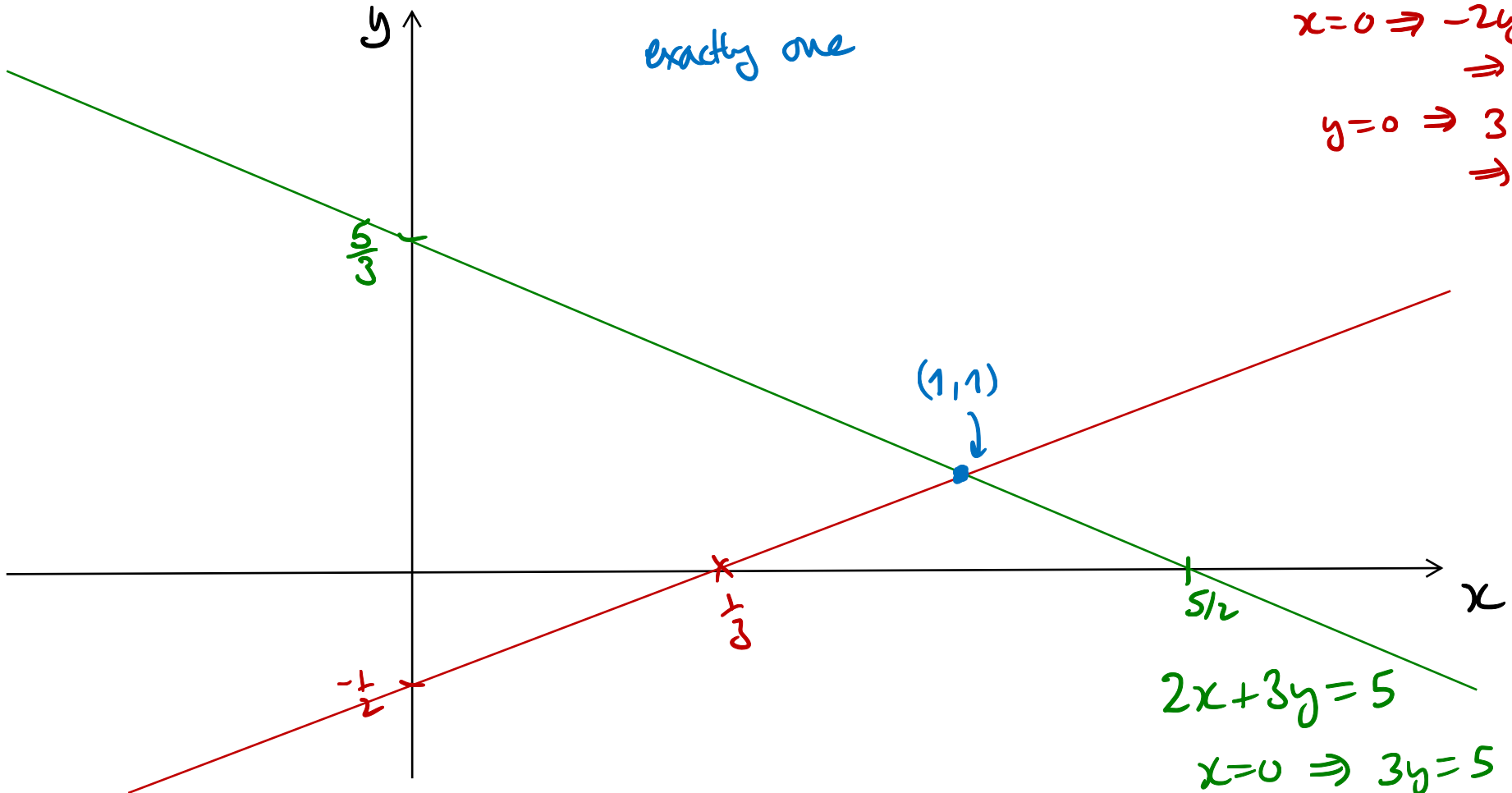
$$g(1.5) = -3 \times 1.5 + 6 = -4.5 + 6 = 1.5$$

Linear equations & their graphs

How many solutions (x, y) are there to $\begin{cases} 3x - 2y = 1 \\ 2x + 3y = 5 \end{cases}$?

exactly one

$$\begin{aligned} 3x - 2y &= 1 \\ x = 0 &\Rightarrow -2y = 1 \\ &\Rightarrow y = -\frac{1}{2} \\ y = 0 &\Rightarrow 3x = 1 \\ &\Rightarrow x = \frac{1}{3} \end{aligned}$$

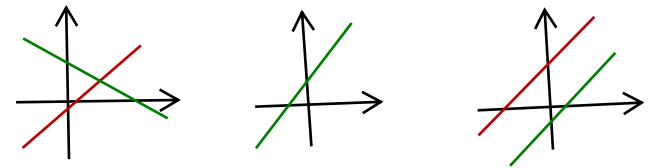


$$2x + 3y = 5$$

$$\begin{aligned} x = 0 &\Rightarrow 3y = 5 \\ &\Rightarrow y = \frac{5}{3} \end{aligned}$$

$$\begin{aligned} y = 0 &\Rightarrow 2x = 5 \\ &\Rightarrow x = \frac{5}{2} \end{aligned}$$

Linear equations & their graphs



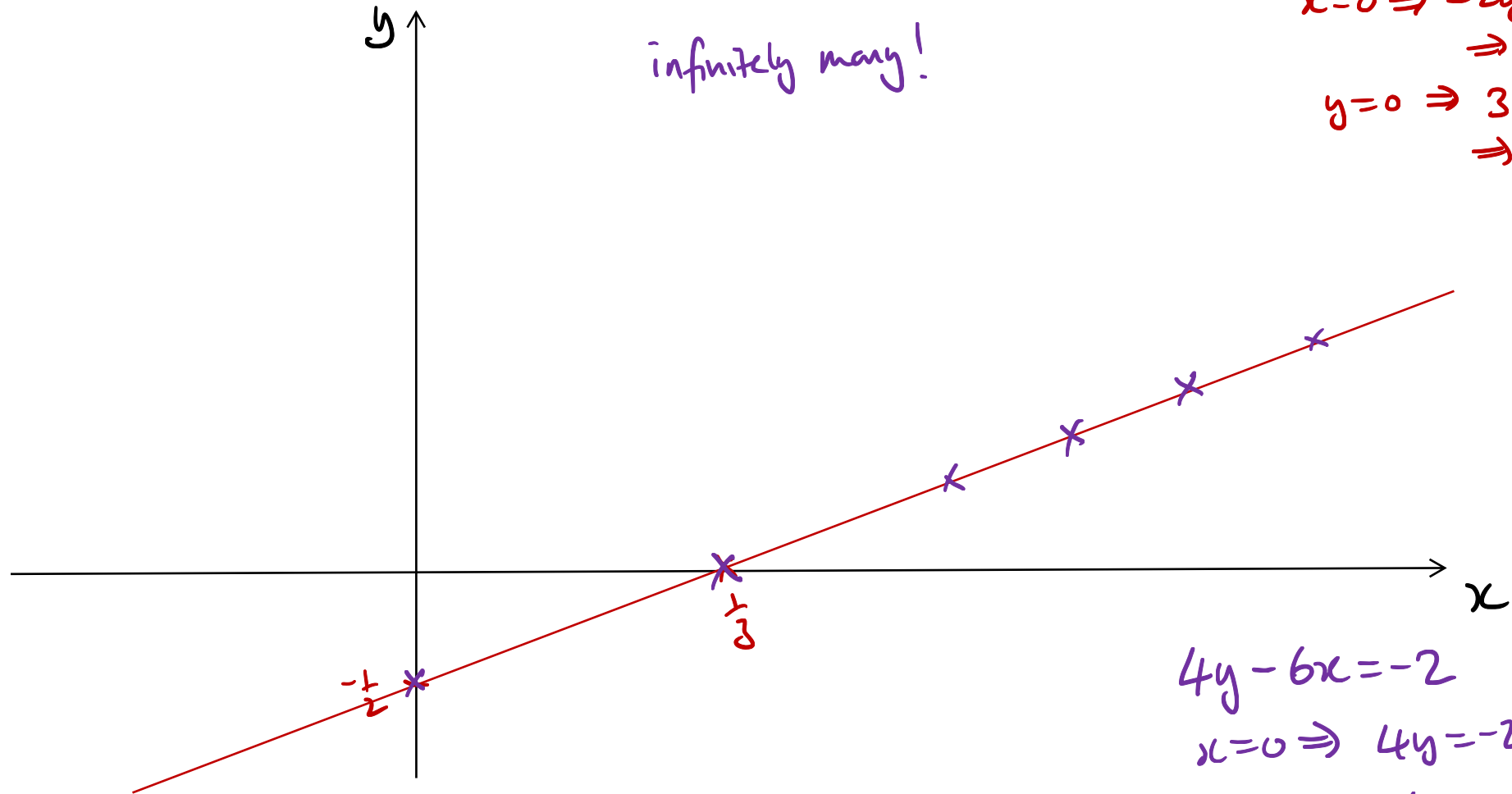
How many solutions (x, y) are there to $\begin{cases} 3x - 2y = 1 \\ 4y - 6x = -2 \end{cases}$? ↖ Same line

infinitely many!

$$3x - 2y = 1$$

$$x = 0 \Rightarrow -2y = 1 \Rightarrow y = -\frac{1}{2}$$

$$y = 0 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$$



$$4y - 6x = -2$$

$$x = 0 \Rightarrow 4y = -2 \Rightarrow y = -\frac{1}{2}$$

$$y = 0 \Rightarrow -6x = -2 \Rightarrow x = \frac{-2}{-6} = \frac{1}{3}$$