Solving Simultaneous Linear Equations by Substitution

Solve
$$\begin{cases} y = 3x - 2 \\ y = x + 2 \end{cases}$$

Solve
$$\begin{cases} 2p - 3q = -2 \\ 4p + q = 24 \end{cases}$$

Find (a, b) where $7a + 2b = 16$ and $-21a - 6b = 24$

Solve
$$\begin{cases} 2x - 3y - 2z = 3 \\ 2y + 3z = 9 \\ z = 5 \end{cases}$$

Solve
$$\begin{cases} 2x + 3y - z = -2 \\ 3x - y + 2z = 13 \\ 5x + 6y + z = 7 \end{cases}$$

$$\boxed{\text{Solving Simultaneous Linear Equations by Substitution}}$$

Solve
$$\begin{cases} y = 3x - 2 & \text{—①} \\ y = x + 2 & \text{—②} \end{cases}$$

② says $y = x + 2$

Sub into ① : $x + 2 = 3x - 2$

$$2 = 2x - 2$$
$$4 = 2x$$
$$\underline{x = 2}$$

So $y = 2 + 2 = \underline{4}$

Soln $\underline{\underline{(2, 4)}}$

Solving Simultaneous Linear Equations by Substitution

Solve
$$\begin{cases} 2p - 3q = -2 & \text{——①} \\ \boxed{4p + q = 24} \end{cases} \iff q = 24 - 4p$$

sub into ①

$$2p - 3(24 - 4p) = -2$$
$$2p - 72 + 12p = -2$$
$$14p - 72 = -2$$
$$14p = -2 + 72$$
$$14p = 70$$
$$p = \frac{70}{14} = 5$$

$$q = 24 - 4 \times 5$$
$$= 24 - 20 = 4$$

$$(p, q) = (5, 4)$$

Solving Simultaneous Linear Equations by Substitution

Find (a,b) where $\quad 7a + 2b = 16 \quad$ and $\quad -21a - 6b = 24$

$$\begin{cases} 7a + 2b = 16 \\ -21a - 6b = 24 \end{cases} \Leftrightarrow \quad 2b = 16 - 7a \quad \Leftrightarrow \quad b = 8 - \frac{7}{2}a$$

$\qquad\qquad\qquad\qquad -21a - 6b = 24 \; - \text{②}$

Sub into ②:

$$-21a - 6\left(8 - \frac{7}{2}a\right) = 24$$

$$-21a - 48 + 21a = 24$$

$$-48 = 24$$

$\underline{\text{no solutions!}}$

inconsistent system.

$-21a - 6b = 24$

$\color{red}{7a + 2b = 16}$
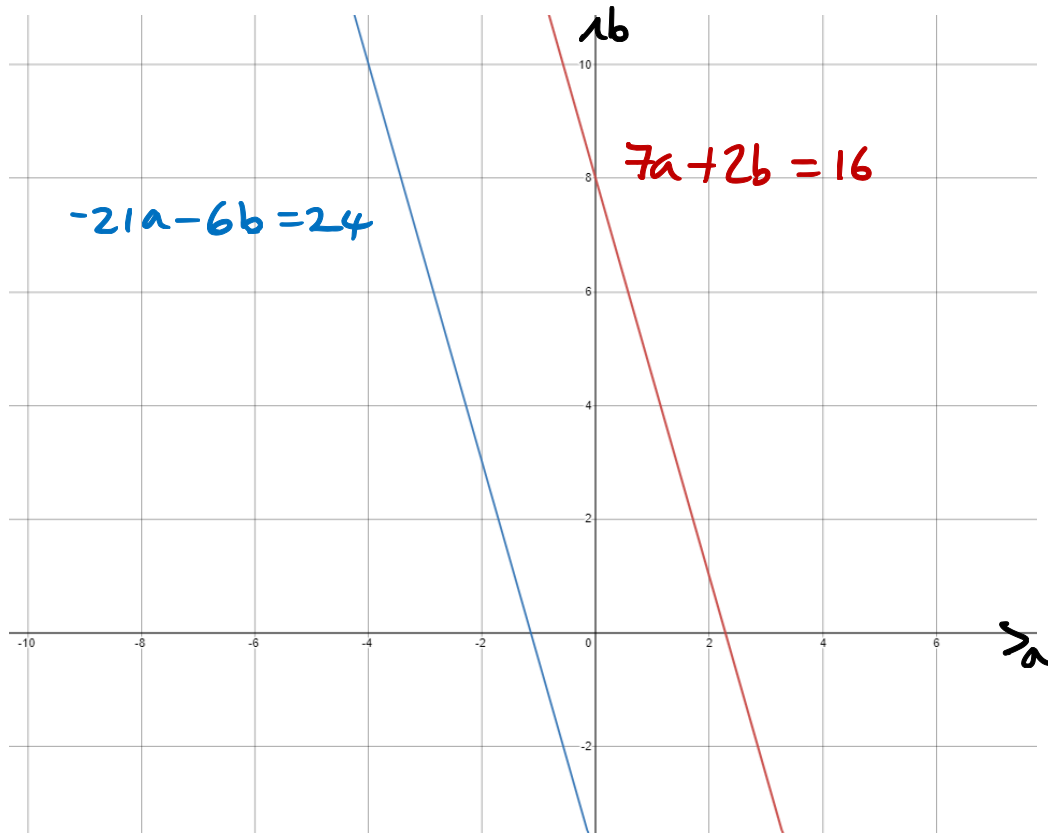
Solve
$$\begin{cases} 2x - 3y - 2z = 3 & —① \\ 2y + 3z = 9 & —② \\ z = 5 & —③ \end{cases}$$

③ : $z = 5$

sub into ②

$2y + 3 \times 5 = 9$

$2y + 15 = 9$

$2y = -6$

$y = -3$

sub into ①

$2x - 3 \times (-3) - 2(5) = 3$

$2x + 9 - 10 = 3$

$2x - 1 = 3$

$2x = 4$

$x = 2$

Solution $(x, y, z) = (2, -3, 5)$

Solving Simultaneous Linear Equations by Substitution

Solve
$$\begin{cases} 2x + 3y - z = -2 & \text{—} \ (1) \\ 3x - y + 2z = 13 & \text{—} \ (2) \\ 5x + 6y + z = 7 & \text{—} \ (3) \end{cases}$$

$\Longleftrightarrow \quad z = 2x + 3y + 2$

- Sub into (2):

$$3x - y + 2(2x + 3y + 2) = 13$$
$$3x - y + 4x + 6y + 4 = 13$$
$$7x + 5y = 9 \quad \text{—} \ (2')$$

- Sub. $z = 2x + 3y + 2$ into (3):

$$5x + 6y + (2x + 3y + 2) = 7$$
$$7x + 9y = 5 \quad \text{—} \ (3')$$

$$7x = 5 - 9y$$

sub $\quad x = \frac{5}{7} - \frac{9}{7}y$ into $(2')$;

$$7\left(\frac{5}{7} - \frac{9}{7}y\right) + 5y = 9$$
$$5 - 9y + 5y = 9$$
$$5 - 4y = 9$$
$$-4y = 4$$
$$y = -1$$

so
$$x = \frac{5}{7} - \frac{9}{7}(-1)$$
$$= \frac{5}{7} + \frac{9}{7} = \frac{14}{7} = 2$$

so $z = 2(2) + 3(-1) + 2$
$$= 4 - 3 + 2$$
$$= 3$$

$\underline{\text{Sol}^n}$ $\quad (x, y, z) = (2, -1, 3)$