Solving Simultaneous Linear Equations by Gaussian Elimination

Find (x, y) where $3x + 2y = 12$ and $5x - 2y = 4$

Solve $\begin{cases} 3x + 2y = 5 \\ 6x - y = 20 \end{cases}$

Solve $\begin{cases} 5x + 7y - 5z = 6 \\ -3x + 2y - 6z = 6 \\ x + 4y - 2z = 8 \end{cases}$

Solve $\begin{cases} 3x_1 + 2x_2 = 5 \\ 4x_2 + 6x_1 = 7 \end{cases}$

Solve $\begin{cases} 6p - 3q + 3r = 0 \\ 3p + q - 6r = -10 \\ 2p + q - 5r = -8 \end{cases}$

Solving Simultaneous Linear Equations by Gaussian Elimination

Find (x, y) where $3x + 2y = 12$ and $5x - 2y = 4$

$$\begin{cases} 3x + 2y = 12 & -① \\ 5x - 2y = 4 & -② \end{cases}$$

$① \to ① + ②$

$$\begin{cases} 8x = 16 \\ 5x - 2y = 4 \end{cases}$$

$① \mapsto \frac{1}{8}①$

$$\begin{cases} x = 2 \\ 5x - 2y = 4 \end{cases}$$

$② \mapsto ② - 5①$

$$\begin{cases} x = 2 \\ -2y = -6 \end{cases}$$

$② \mapsto -\frac{1}{2}②$

$$\begin{cases} x = 2 \\ y = 3 \end{cases}$$

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Solve $\begin{cases} 3x + 2y = 5 & -(1) \\ 6x - y = 20 & -(2) \end{cases}$

$(2) \mapsto (2) - 2(1)$ $\begin{cases} 3x + 2y = 5 \\ -5y = 10 \end{cases}$

$(2) \mapsto -\frac{1}{5}(2)$ $\begin{cases} 3x + 2y = 5 \\ y = -2 \end{cases}$

$(1) \to (1) - 2(2)$ $\begin{cases} 3x = 9 \\ y = -2 \end{cases}$

$(1) \mapsto \frac{1}{3}(1)$ $\begin{cases} x = 3 \\ y = -2 \end{cases}$

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Solve $\begin{cases} 5x + 7y - 5z = 6 & -① \\ -3x + 2y - 6z = 6 & -② \\ x + 4y - 2z = 8 & -③ \end{cases}$

$①\mapsto ① - 5③$, $②\mapsto ② + 3①$ $\Rightarrow$ $\begin{cases} -13y + 5z = -34 \\ +14y - 12z = 30 \\ x + 4y - 2z = 8 \end{cases}$

$②\mapsto ② + ①$ $\Rightarrow$ $\begin{cases} -13y + 5z = -34 \\ y - 7z = -4 \\ x + 4y - 2z = 8 \end{cases}$

$①\mapsto ① + 13②$, $③\mapsto ③ - 4②$ $\Rightarrow$ $\begin{cases} -86z = -86 \\ y - 7z = -4 \\ x + 26z = 24 \end{cases}$

$①\to \frac{-1}{86}①$ $\Rightarrow$ $\begin{cases} z = 1 \\ y - 7z = -4 \\ x + 26z = 24 \end{cases}$

$②\mapsto ② + 7①$, $③\to ③ - 26①$ $\Rightarrow$ $\begin{cases} z = 1 \\ y = 3 \\ x = -2 \end{cases}$

Soln $(x, y, z) = (-2, 3, 1)$

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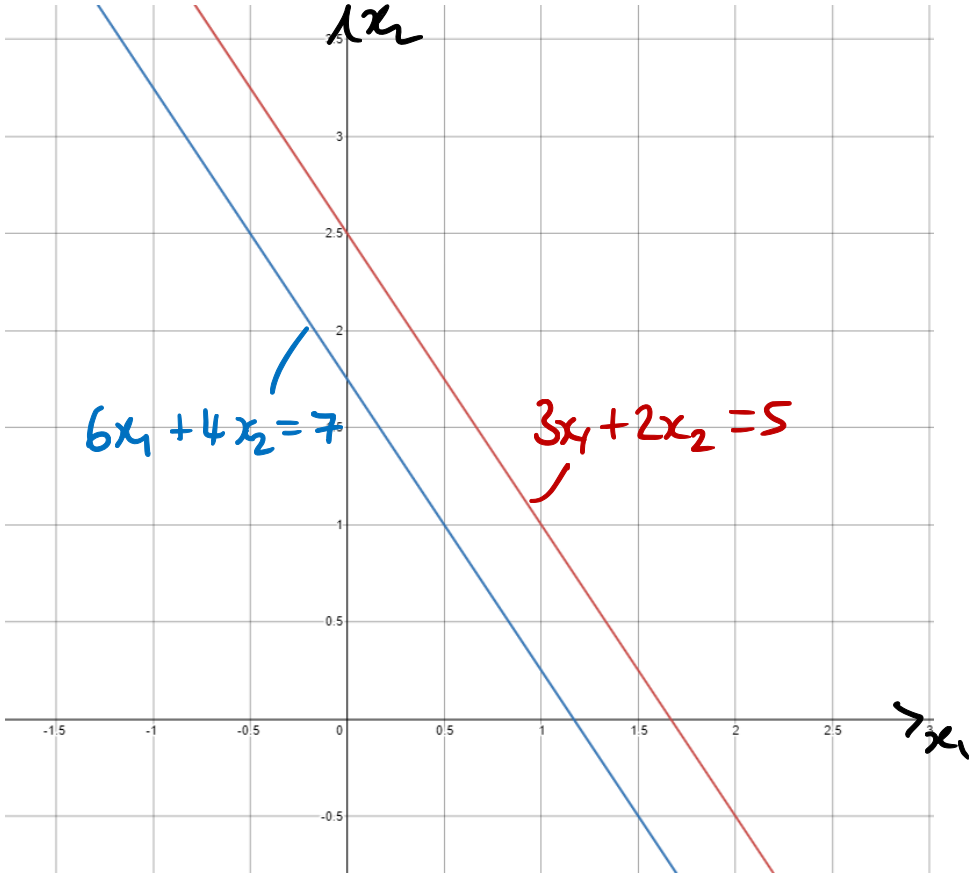
Solve $\begin{cases} 3x_1 + 2x_2 = 5 \\ 4x_2 + 6x_1 = 7 \end{cases}$

$$\begin{cases} 3x_1 + 2x_2 = 5 & -① \\ 6x_1 + 4x_2 = 7 & -② \end{cases} \xrightarrow{② \to ② - 2①} \begin{cases} 3x_1 + 2x_2 = 5 \\ 0 = -3 \end{cases}$$

no solutions

(inconsistent system)



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Solve $\begin{cases} 6p - 3q + 3r = 0 & -① \\ 3p + q - 6r = -10 & -② \\ 2p + q - 5r = -8 & -③ \end{cases}$

$① \mapsto ① + 3②$, $③ \mapsto ③ - ②$ $\begin{cases} 15p \quad -15r = -30 \\ 3p + q - 6r = -10 \\ -p \quad + r = 2 \end{cases}$

$① \mapsto \frac{1}{15}①$ $\begin{cases} p \quad - r = -2 \\ 3p + q - 6r = -10 \\ -p \quad + r = 2 \end{cases}$

$② \mapsto ② - 3①$ $\begin{cases} p \quad - r = -2 \\ q - 3r = -4 \\ -p \quad + r = 2 \end{cases}$

$③ \mapsto ③ + ①$ $\begin{cases} p - r = -2 \\ q - 3r = -4 \\ 0 = 0 \end{cases}$

$p = -2 + r$

$q = -4 + 3r$

$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -2+r \\ -4+3r \\ r \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -4 \\ 0 \end{pmatrix} + r \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$

Infinitely many solutions

eg. $r = 0$, $p = -2$, $q = -4$

$r = 1$, $p - 1 = -2$, $q - 3 = -4$

$p = -1$, $q = 1$