Product Rule for Differentiation: $(fg)' = f'g + fg'$

$\frac{d}{dy}\left(3y^2 e^{2y} \right)$

$\pi(q) = 3q^{\frac{2}{3}} + 2, \quad \omega(q) = 2q^{3/5} - 1,$

$(\pi\omega)'(q) = ?$

$\frac{d}{dx}\left((x^2+3)\ln(5x) \right)$

$\frac{d}{dt}\left(4t^3 e^{5t} \ln(2t) \right)$

$\frac{d}{dt}\left(t^3 (t^2+2) \right)$

Calculate $\frac{d}{dx}\left(\frac{1}{f(x)} \right)$ in terms of $\frac{df}{dx}$.

$$\boxed{\text{Product Rule for Differentiation:} \qquad (fg)' = f'g + fg'}$$

multiplication

$$(f \times g)' = (f' \times g) + (f \times g')$$

$$\frac{d}{dx}\left(f(x)\,g(x)\right) = \frac{df}{dx}\,g(x) + f(x)\,\frac{dg}{dx}$$

Product Rule for Differentiation: $\quad (fg)' = f'g + fg'$

$$\frac{d}{dy}\Big(\underbrace{3y^2}_{f(y)}\ \underbrace{e^{2y}}_{g(y)}\Big) = f'(y)\,g(y) + f(y)\,g'(y) = 6y\,e^{2y} + 3y^2\,2e^{2y}$$

$$= \underline{\underline{6ye^{2y}(1+y)}}$$

$f'(y) = 6y \qquad g'(y) = 2e^{2y}$

$$\frac{d}{dx}\Big(\underbrace{(x^2+3)}_{f(x)}\ \underbrace{\ln(5x)}_{g(x)}\Big) = f'(x)\,g(x) + f(x)\,g'(x) = 2x\ln(5x) + (x^2+3)\frac{1}{x}$$

$$= 2x\ln(5x) + x + \frac{3}{x}$$

$f'(x) = 2x \qquad g'(x) = \frac{1}{x}$

$$\frac{d}{dt}\Big(\underline{t^3}\,\underline{(t^2+2)}\Big) = 3t^2(t^2+2) + t^3(2t) = 3t^4 + 6t^2 + 2t^4 = \underline{5t^4 + 6t^2}$$

$$\text{''}\quad \frac{d}{dt}\left(t^5 + 2t^3\right) = \underline{5t^4 + 6t^2}$$

Product Rule for Differentiation: $\quad (fg)' = f'g + fg'$

$$\pi(q) = 3q^{\frac{2}{3}} + 2, \quad \omega(q) = 2q^{3/5} - 1,$$

$$(\pi\omega)'(q) = \pi'(q)\,\omega(q) + \pi(q)\,\omega'(q)$$

$$= 2q^{-\frac{1}{3}}\left(2q^{\frac{3}{5}} - 1\right) + \left(3q^{2/3} + 2\right)\left(\tfrac{6}{5}\, q^{-\frac{2}{5}}\right)$$

$$= 4q^{4/15} - 2q^{-\frac{1}{3}} + \tfrac{18}{5} q^{\frac{4}{15}} + \tfrac{12}{5} q^{-2/5}$$

$$-\tfrac{1}{3} + \tfrac{3}{5} = \tfrac{-5}{15} + \tfrac{9}{15} = \tfrac{4}{15}$$

$$\tfrac{2}{3} - \tfrac{2}{5} = \tfrac{10}{15} - \tfrac{6}{15} = \tfrac{4}{15}$$

$$\frac{d}{dt}\left(\underline{4t^3}\ \underline{e^{5t}\ln(2t)}\right) = \frac{d}{dt}(4t^3)\, e^{5t}\ln(2t) + 4t^3 \frac{d}{dt}\left(\underline{e^{5t}}\,\underline{\ln(2t)}\right)$$

$$= 12t^2 e^{5t}\ln(2t) + 4t^3\left(\frac{d}{dt}(e^{5t})\ln(2t) + e^{5t}\frac{d}{dt}(\ln(2t))\right)$$

$$= 12t^2 e^{5t}\ln(2t) + 4t^3\left(5e^{5t}\ln(2t) + e^{5t}\tfrac{1}{t}\right)$$

$$= \underbrace{12}_{3}\underbrace{t^2}_{1}\underline{e^{5t}}\ln(2t) + \underbrace{20}_{5}\underbrace{t^3}_{t}\underline{e^{5t}}\ln(2t) + \underbrace{4}_{1}\underbrace{t^2}_{1}\underline{e^{5t}})$$

$$= 4t^2 e^{5t}\left(3\ln(2t) + 5t\ln(2t) + 1\right)$$

$$\boxed{\text{Product Rule for Differentiation:} \qquad (fg)' = f'g + fg'}$$

Calculate $\frac{d}{dx}\left(\frac{1}{f(x)}\right)$ in terms of $\frac{df}{dx}$.

$$1 = f(x)\,\frac{1}{f(x)}$$

product rule

$$0 = \frac{d}{dx}(1) = \frac{d}{dx}\left(f(x)\,\frac{1}{f(x)}\right) = \frac{df}{dx}\,\frac{1}{f(x)} + f(x)\,\frac{d}{dx}\left(\frac{1}{f(x)}\right)$$

$$-\frac{df}{dx}\,\frac{1}{f(x)} = f(x)\,\frac{d}{dx}\left(\frac{1}{f(x)}\right)$$

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right) = -\frac{df}{dx}\,\frac{1}{(f(x))^2}$$