Chain Rule for Differentiation: $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\, g'(x)$

$\frac{d}{dx}\left((3x-1)^2\right)$

Also known as:

- The function-of-a-function rule
- The composition rule

$\frac{d}{dy}\left((y+3)^{200}\right)$

$\frac{d}{dx}\left(e^{3x^2+2}\right)$

$\frac{d}{dt}\left(\ln(2t^3+4e^{5t})\right)$

Calculate the derivative of f^{-1} in terms of f'

$(3x-1)^2 = 9x^2 - 6x + 1$

$\frac{d}{dx}\left((3x-1)^2\right) = 18x - 6$

Chain Rule for Differentiation: $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\,g'(x)$

$$\frac{d}{dx}\left((3x-1)^2\right) = \frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\,g'(x) = 2(g(x))\,g'(x)$$

$g(x) = 3x-1$, $g'(x) = 3$

$f(t) = t^2$, $f'(t) = 2t$

$$= 2(3x-1)\,3 = 18x - 6$$

$$\frac{d}{dy}\left((y+3)^{200}\right) = 200(y+3)^{199}\,1 = 200(y+3)^{199}$$

$g(y) = y+3$, $g'(y) = 1$

$f(z) = z^{200}$, $f'(z) = 200z^{199}$

$$\frac{d}{dx}\left(e^{(3x^2+2)}\right) = e^{(3x^2+2)}\,6x = \underline{\underline{6xe^{3x^2+2}}}$$

$g(x) = 3x^2+2$, $g'(x) = 6x$

$f(t) = e^t$, $f'(t) = e^t$

$$\boxed{\text{Chain Rule for Differentiation: } \quad \frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\, g'(x)}$$

$$\frac{d}{dt}\left(\ln(2t^3+4e^{5t})\right) = \frac{1}{(2t^3+4e^{5t})}\, 6t^2+20e^{5t} = \frac{6t^2+20e^{5t}}{2t^3+4e^{5t}}$$

$$g(t) = 2t^3+4e^{5t}, \quad g'(t) = 6t^2+20e^{5t}$$

$$f(y) = \ln(y), \quad f'(y) = \frac{1}{y}$$

$$\frac{d}{dt}\left(e^{((3\ln t)^2)}\right) = f'(g(t))\, g'(t) = e^{((3\ln t))^2)}\, \frac{18}{t}\ln(t) = \frac{18}{t}\ln(t)\, e^{(3\ln(t))^2}$$

$$g(t) = (3\ln(t))^2 = (3\ln(t))^2 = k(h(t))$$

$$h(t) = 3\ln(t)$$

$$h'(t) = \frac{3}{t}$$

$$k(z) = z^2$$

$$k'(z) = 2z$$

$$f(y) = e^y, \quad f'(y) = e^y$$

$$g'(t) = k'(h(t))\, h'(t)$$

$$= 2(3\ln(t))\, \frac{3}{t} = \frac{18}{t}\ln(t)$$

$$\boxed{\text{Chain Rule for Differentiation:} \quad \frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\,g'(x)}$$

Calculate the derivative of f^{-1} in terms of f'

let $g(x) = f^{-1}(x)$

$f^{-1}(f(x)) = x$

$f(f^{-1}(x)) = x$

$$x = f(g(x))$$

chain rule

$$1 = \frac{d}{dx}(x) = \frac{d}{dx}(f(g(x))) = f'(g(x))\,g'(x)$$

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{f'(f^{-1}(x))}$$

$$\underline{\underline{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}}$$