Combining differentiation rules

Power Rule $\frac{d}{dx}(x^n) = nx^{n-1}$

exponential $\frac{d}{dx}(e^{ax}) = ae^{ax}$

logarithms $\frac{d}{dx}(\ln(ax+b)) = \frac{a}{ax+b}$

Sum Rule $(af+bg)' = af' + bg'$

Product Rule $(fg)' = f'g + fg'$

Quotient Rule $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

Chain Rule $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\,g'(x)$

$$\frac{d}{dx}\left(\ln(3x^2+2e^{5x})\right)$$

$$\frac{d}{dy}\left(\frac{\ln(2y)+3y}{\sqrt{y^2+2}}\right)$$

$$\frac{d}{dt}\left(\frac{t^3e^{4t}}{4t^2+\ln(5t)}\right)$$

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$$\frac{d}{dx}\left(\ln(3x^2+2e^{5x})\right) \underset{\text{rule}}{\overset{\text{chain}}{=}} \frac{1}{(3x^2+2e^{5x})} \frac{d}{dx}\left(3x^2+2e^{5x}\right)$$

$$\underset{\text{rule}}{\overset{\text{sum}}{=}} \frac{1}{3x^2+2e^{5x}} \left(3\frac{d}{dx}(x^2) + 2\frac{d}{dx}(e^{5x})\right)$$

$$= \frac{1}{3x^2+2e^{5x}} \left(6x + 10e^{5x}\right)$$

$$= \frac{6x+10e^{5x}}{3x^2+2e^{5x}}$$

$$\frac{d}{dx}\left(\ln(g(x))\right) \underset{\text{rule}}{\overset{\text{chain}}{=}} \frac{1}{g(x)} g'(x)$$

$$= \frac{g'(x)}{g(x)}$$

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$$\frac{d}{dt}\left(\frac{t^3 e^{4t}}{4t^2 + \ln(5t)}\right) \underset{\text{rule}}{\overset{\text{quotient}}{=}} \frac{u'v - uv'}{v^2} = \frac{(3+4t)t^2e^{4t}\left(4t^2 + \ln(5t)\right) - t^3e^{4t}\left(8t + \frac{1}{t}\right)}{\left(4t^2 + \ln(5t)\right)}$$

$u = t^3 e^{4t}$

$$u' = \frac{d}{dt}(t^3 e^{4t}) \underset{\text{rule}}{\overset{\text{product}}{=}} 3t^2 e^{4t} + t^3 4e^{4t} = (3+4t)t^2 e^{4t}$$

$v = 4t^2 + \ln(5t)$

$$v' \underset{\text{sum rule}}{=} 8t + \frac{1}{t}$$

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$$\frac{d}{dy}\left(\frac{\ln(2y)+3y}{\sqrt{y^2+2}}\right) \overset{\text{quotient}}{\underset{\text{rule}}{=\!=}} \frac{u'v - uv'}{v^2}$$

$$= \frac{\overset{1+3y}{\cancel{(1/y+3)}}\,\cancel{\pm}\sqrt{y^2+2} - (\ln(2y)+3y)\frac{y^2}{\cancel{\sqrt{y^2+2}}}}{y\left(\sqrt{y^2-2}\right)^{\cancel{2/4}\;3}}$$

$$= \frac{(1+3y)(y^2+2) - (\ln(2y)+3y)\,y^2}{y(y^2+2)^{3/2}} = \frac{y^2+2+6y-y^2\ln(2y)}{y(y^2+2)^{3/2}}$$

$u = \ln(2y) + 3y$

$u' \overset{\text{sum}}{\underset{\text{rule}}{=\!=}} \frac{1}{y} + 3$

$v = \sqrt{y^2+2} = \sqrt{y^2+2}$,

$v' \overset{\text{chain}}{\underset{\text{rule}}{=\!=}} f'(g(y))\,g'(y) = \frac{1}{2\sqrt{y^2+2}}\,2y = \frac{y}{\sqrt{y^2+2}}$

$g(y) = y^2+2$, $g'(y) = 2y$

$f(t) = \sqrt{t} = t^{1/2}$, $f'(t) = \frac{1}{2}t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$

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$$\underset{\text{sum}}{=} \frac{1}{3x^2+2e^{5x}} \, 6x + 10e^{5x} = \frac{6x+10e^{5x}}{3x^2+2e^{5x}}$$

Rule

$$\frac{d}{dx} \ln(g(x)) \overset{\text{chain}}{=} \frac{1}{g(x)} g'(x) = \underline{\underline{\frac{g'(x)}{g(x)}}}$$

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$$\frac{d}{dt}\left(4t^2 + \ln(5t)\right) = 8t + \frac{1}{t}$$

$| \, 8n$

$$\frac{d}{dt}\left(\frac{t^3 e^{4t}}{4t^2 + \ln(5t)}\right) \overset{\text{quotient}}{=} \frac{\frac{d}{dt}(t^3 e^{4t})\left(4t^2 + \ln(5t)\right) - t^3 e^{4t}\frac{d}{dt}\left(4t^2 + \ln(5t)\right)}{\left(4t^2 + \ln(5t)\right)^2}$$

$$\frac{d}{dt}(t^3 e^{4t}) \overset{\text{prod}}{=} 3t^2 e^{4t} + t^3 4e^{4t}$$

$$= (3 + 4t)\, t^2 e^{4t}$$

$$= \frac{(3+4t)\, t^2 e^{4t}\left(4t^2 + \ln(5t)\right) - t^3 e^{4t}\left(8t + \frac{1}{t}\right)}{\left(4t^2 + \ln(5t)\right)^2}$$