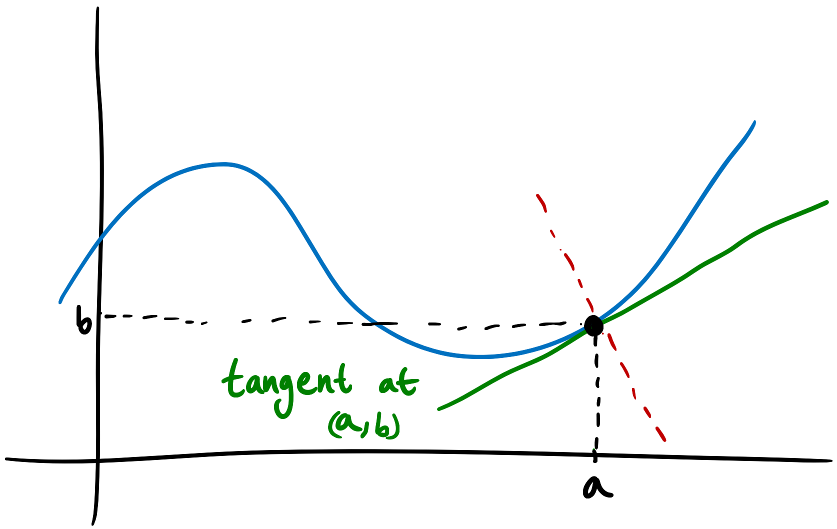
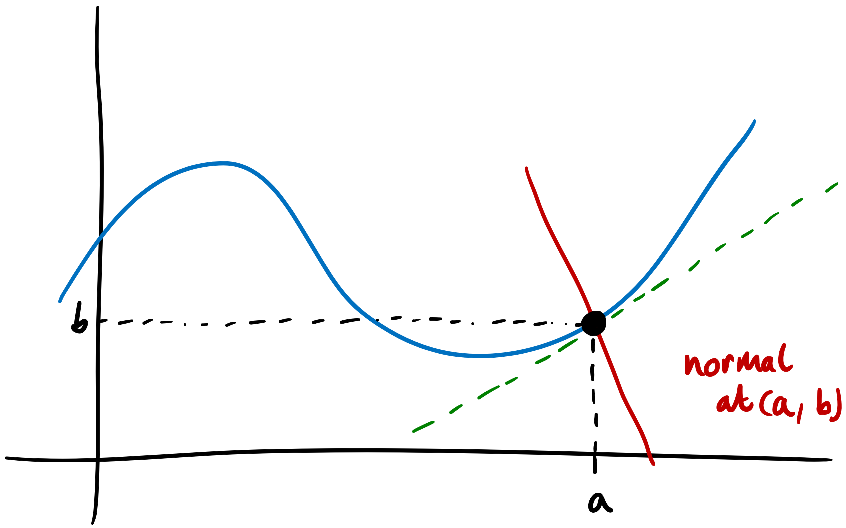
Tangents and Normals

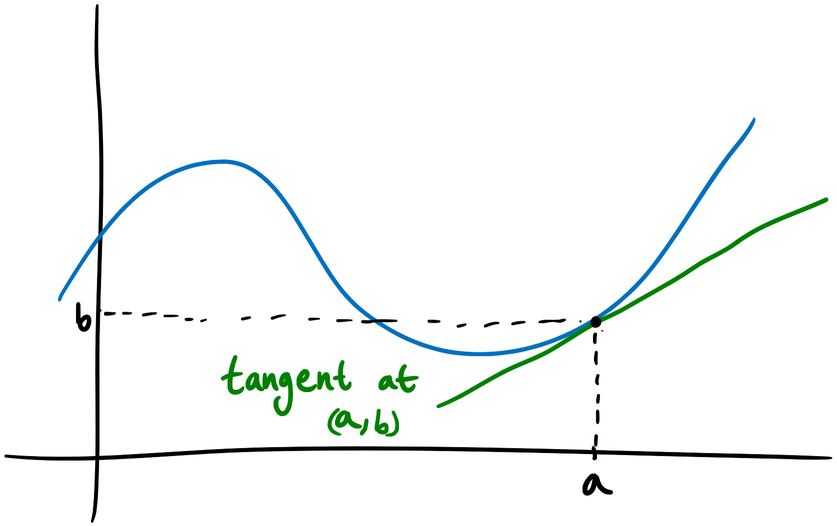


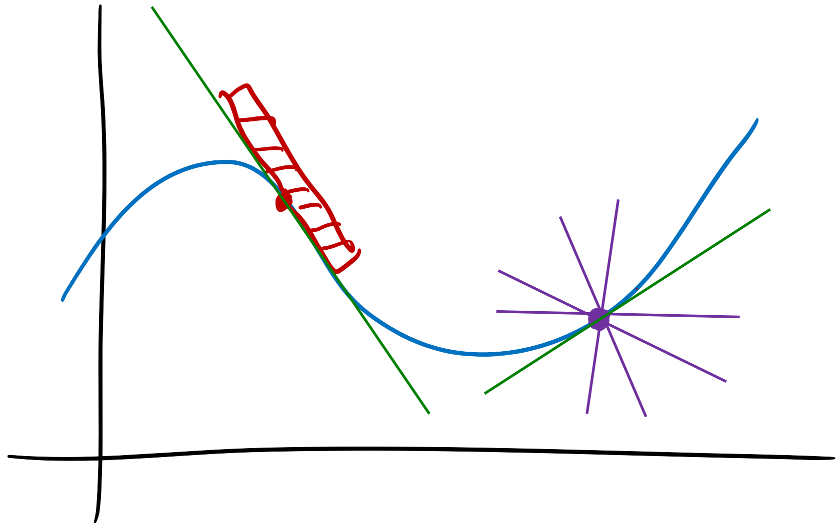


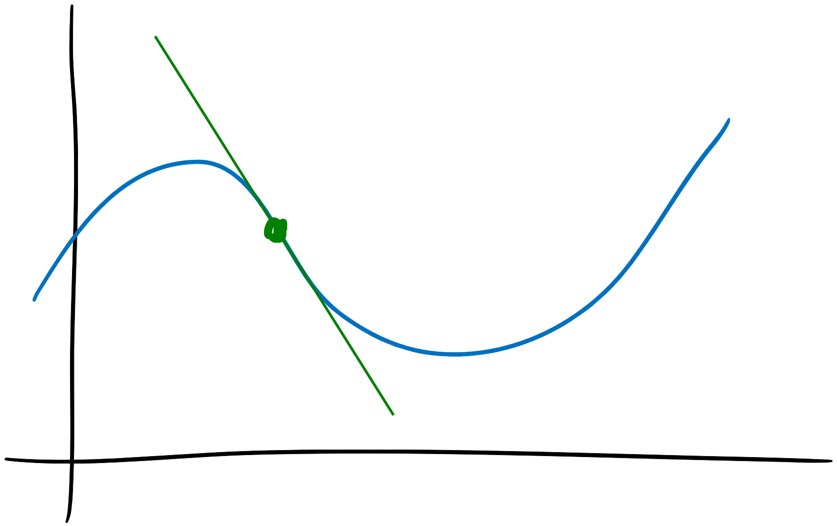
Find the tangent and normal to the curve $y = 3x^2 + 2x - 5$ at the point where $x = -2$

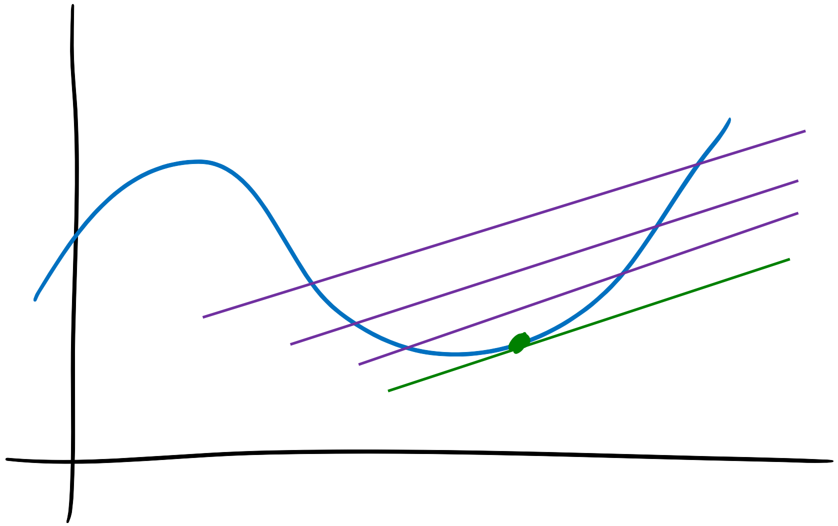
Find the tangent and normal to the curve $y = \frac{x^3}{8} - 3\sqrt{x}$ at the point when $x = 4$

Tangents

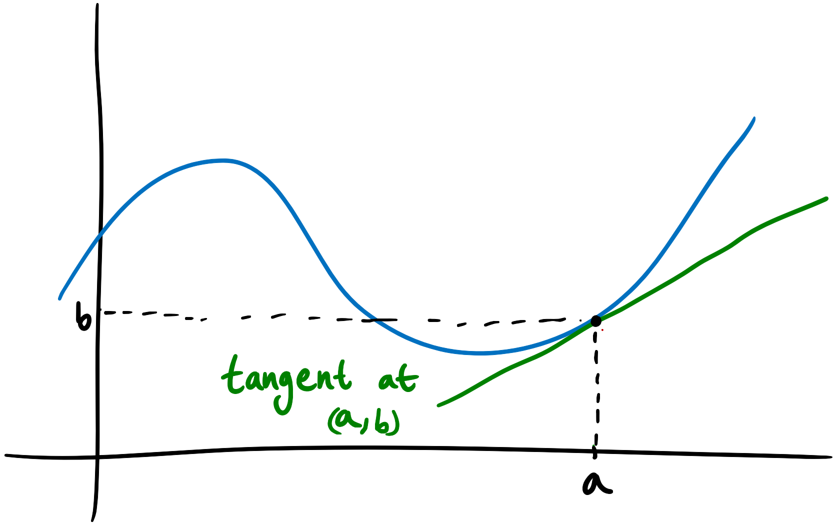


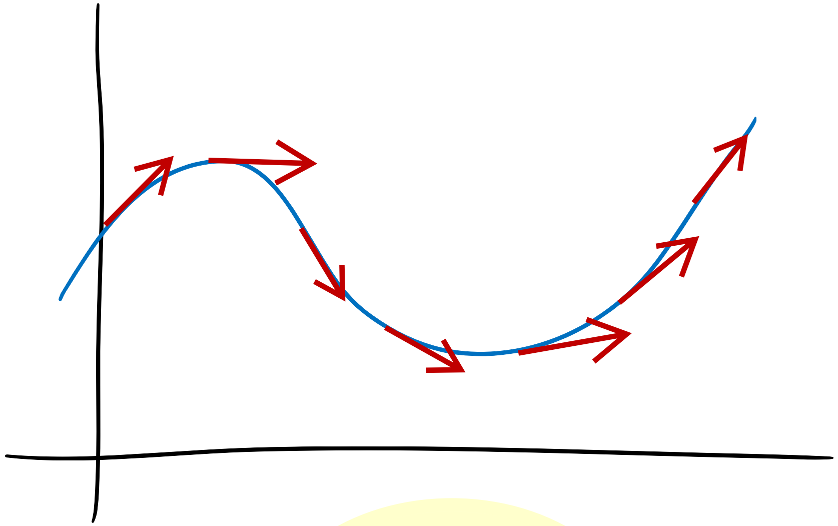


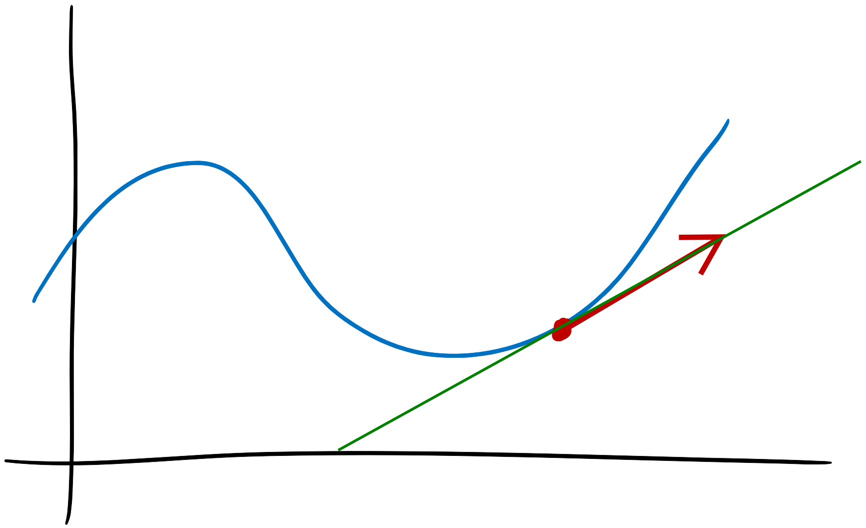


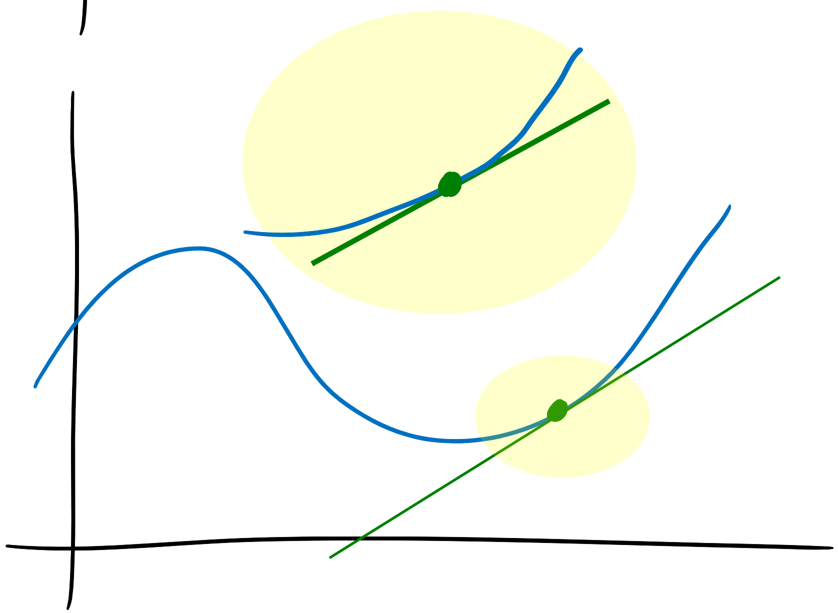


Tangents







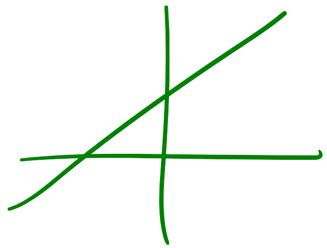


Tangents

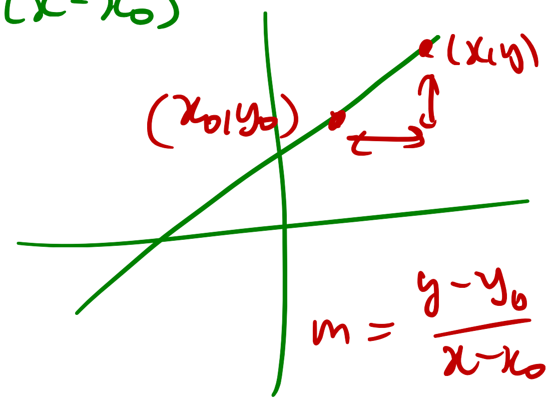
The **tangent** of $y=f(x)$ at $(a, f(a))$ is the

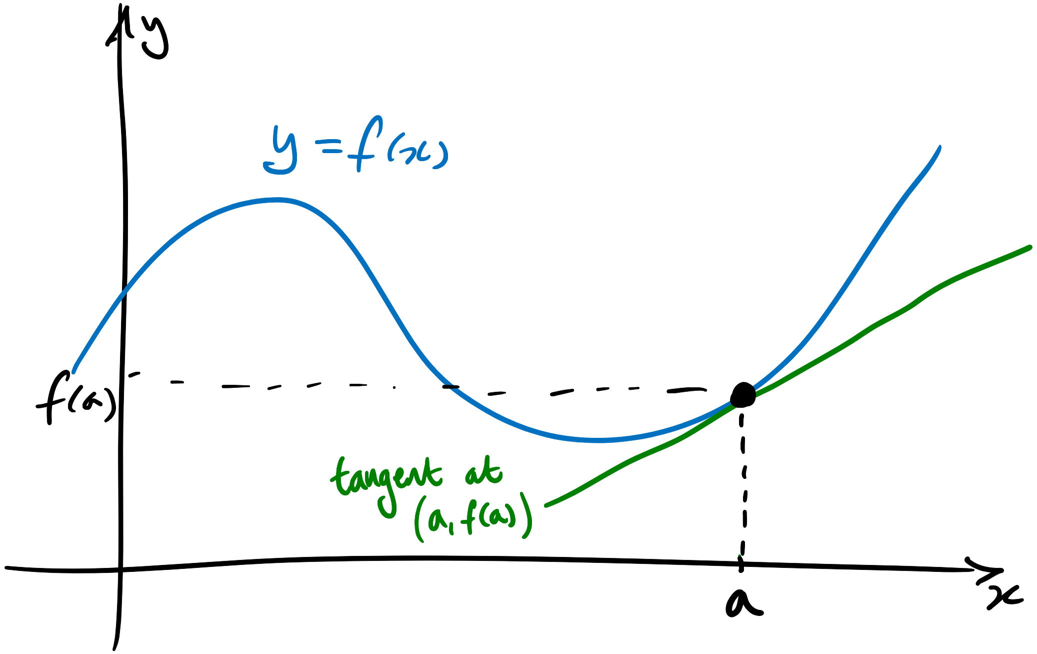
- straight line
- that passes through $(a, f(a))$
- whose slope is $\underbrace{f'(a)}_{m}$

$y = mx + c$



$y - y_0 = m(x - x_0)$

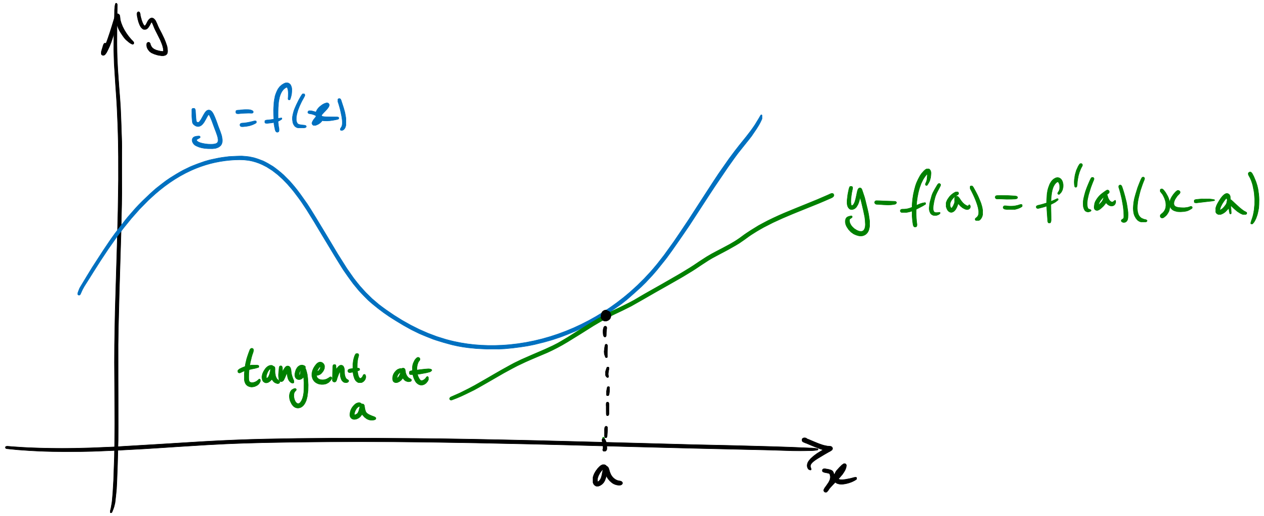




$y - y_0 = m(x - x_0)$

$$\boxed{y - f(a) = f'(a)(x - a)}$$

Tangents



Find the tangent to the curve

$$y = 3x^2 + 2x - 5$$

at the point where $x = -2$

When $x = -2$, $\quad y = 3(-2)^2 + 2(-2) - 5$

$$= 12 - 4 - 5$$

$$= 3$$

$$\frac{dy}{dx} = 6x + 2$$

so at $x = -2$, $\quad \frac{dy}{dx} = 6 \times (-2) + 2$

$$= -12 + 2 \quad = -10$$

Tangent line

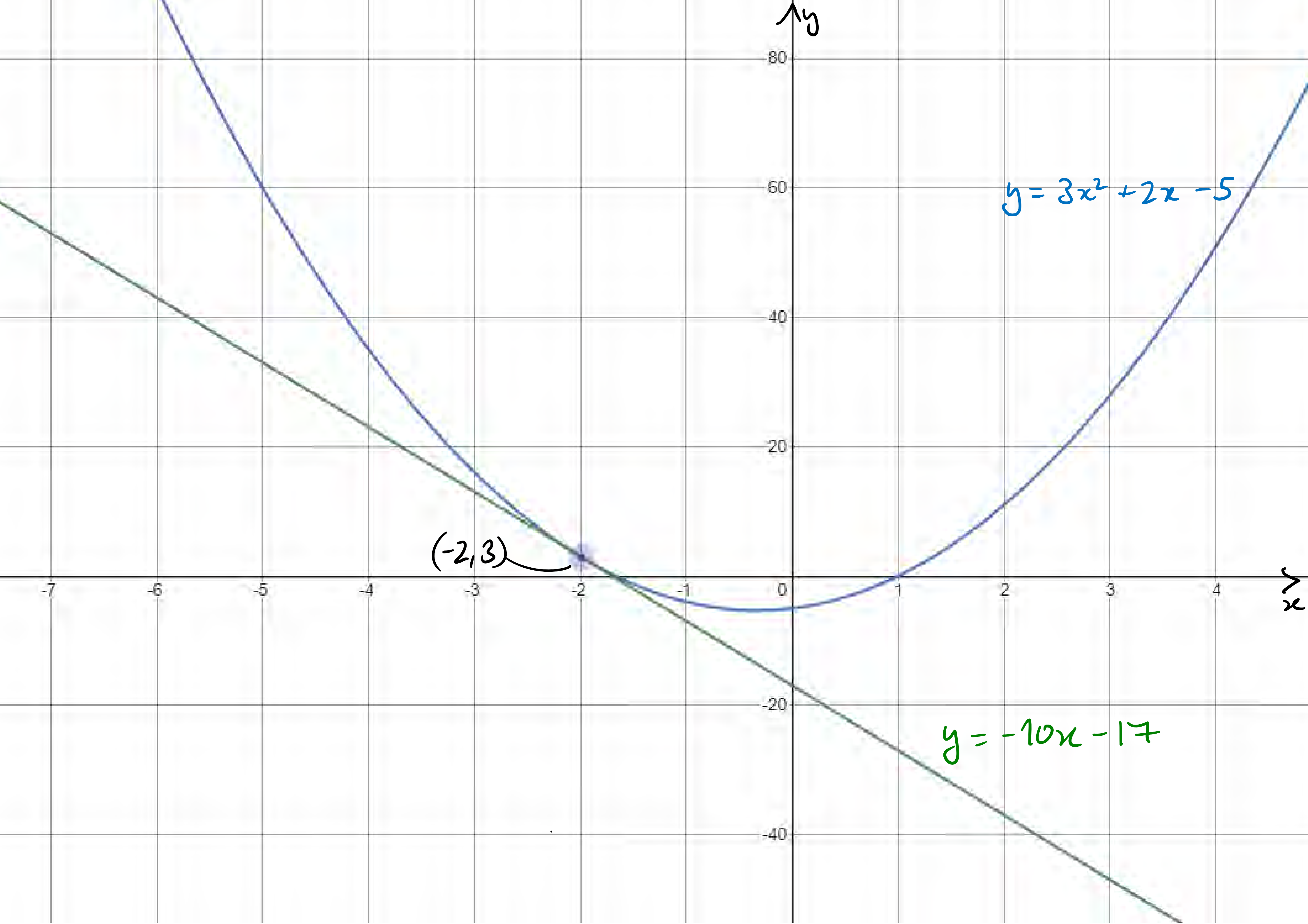
$$y - y_0 = m(x - x_0)$$

$$y - 3 = -10(x - (-2))$$

$$y - 3 = -10x - 20$$

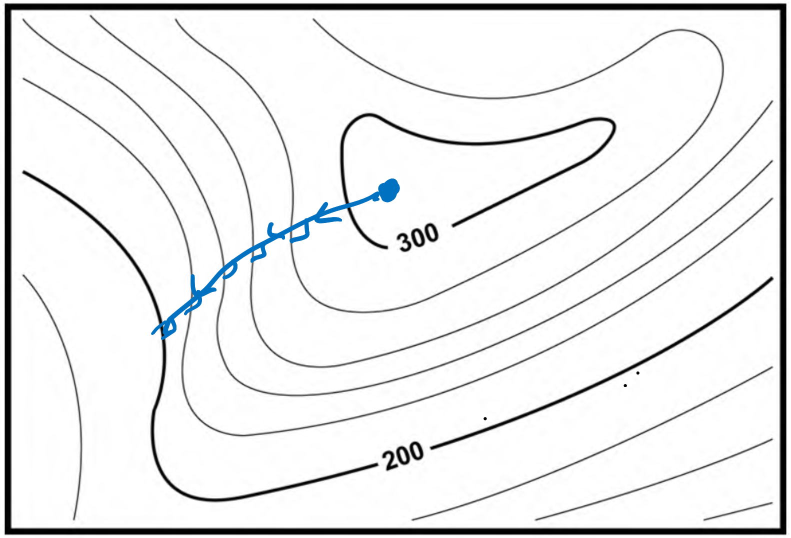
$$y = -10x - 20 + 3$$

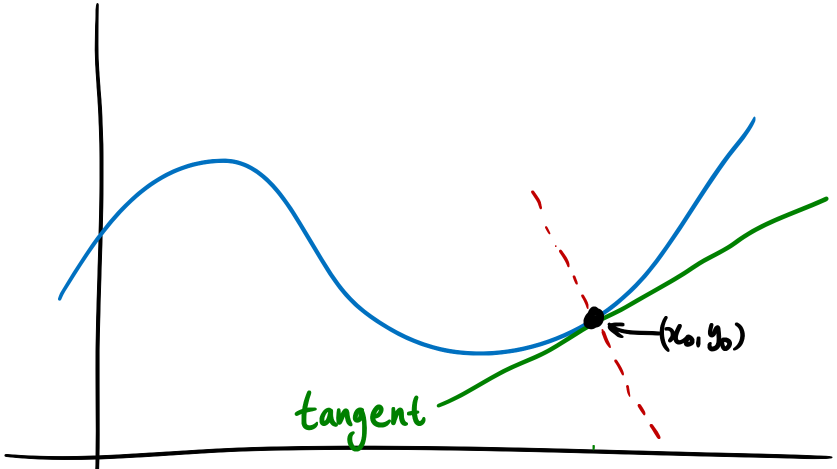
$$y = -10x - 17$$



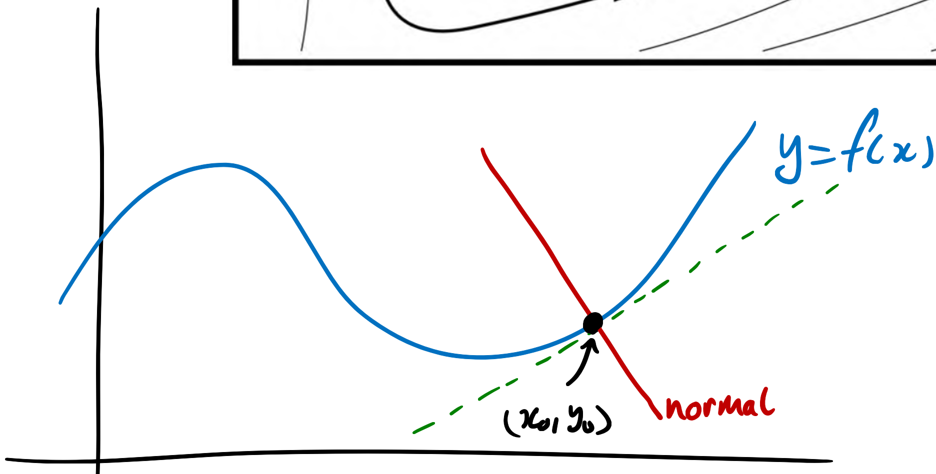
y
x
80
60
40
20
-20
-40
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
$y = 3x^2 + 2x - 5$
$y = -10x - 17$
(-2, 3)

Tangents and Normals

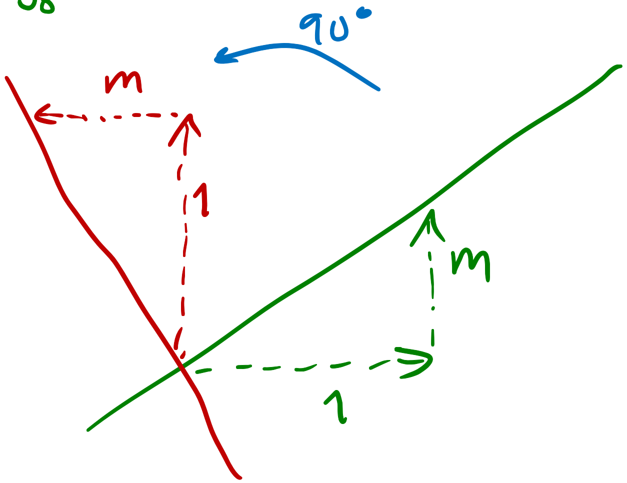




$$y - y_0 = m(x - x_0)$$



$$y - y_0 = n(x - x_0)$$



$$n = \frac{1}{-m} = \frac{-1}{m} \quad , \quad \underline{nm = -1}$$

$$m = f'(x_0) \qquad n = \frac{-1}{f'(x_0)}$$

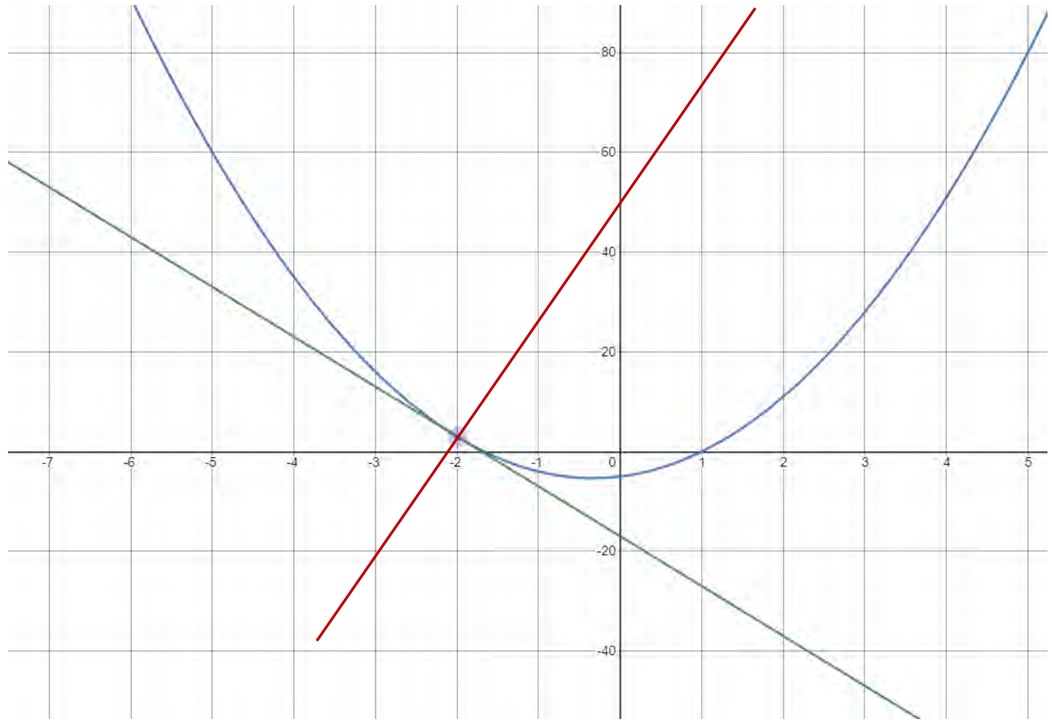
$$y - f(x_0) = \frac{-1}{f'(x_0)}(x - x_0)$$

Tangents and Normals

Find the tangent to the curve

$$y = 3x^2 + 2x - 5$$

at the point where $x = -2$



When $x = -2$, $\quad y = 3(-2)^2 + 2(-2) - 5$

$$= 12 - 4 - 5$$

$$= 3$$

$$\frac{dy}{dx} = 6x + 2$$

So at $x = -2$, $\quad \frac{dy}{dx} = 6 \times (-2) + 2$

$$= -12 + 2 = -10$$

<u>Tangent line</u>

$$y - y_0 = m(x - x_0)$$

$$y - 3 = -10(x - (-2))$$

$$y - 3 = -10x - 20$$

$$y = -10x - 20 + 3$$

$$\underline{\underline{y = -10x - 17}}$$

<u>Normal</u>

$$y - y_0 = n(x - x_0)$$

$$y - 3 = \frac{-1}{-10}(x - (-2))$$

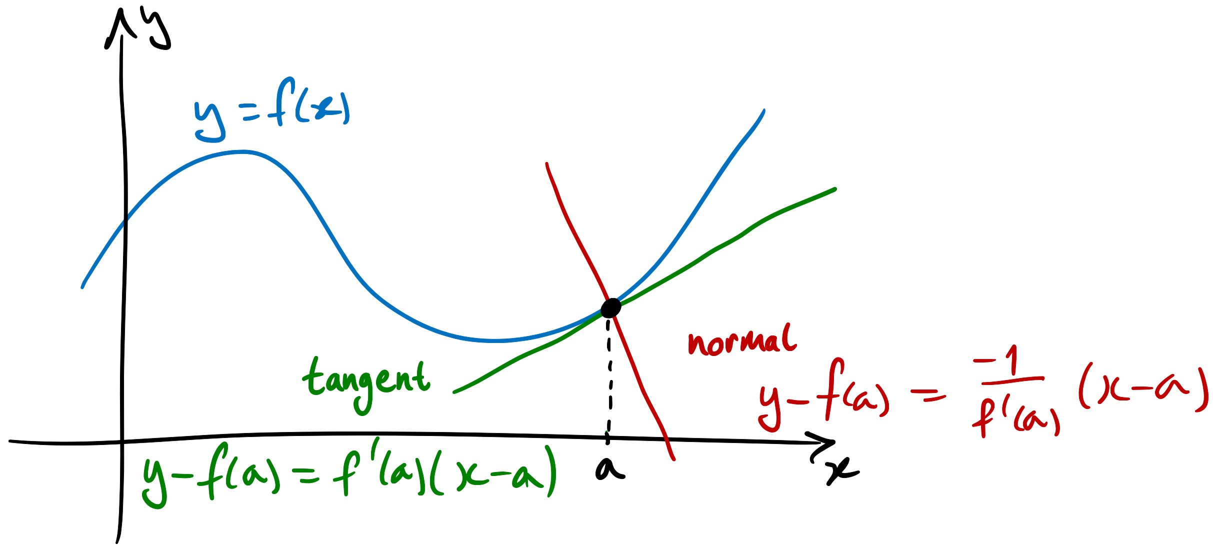
$$y - 3 = \frac{1}{10}(x + 2)$$

$$y - 3 = \frac{x}{10} + \frac{1}{5}$$

$$y = \frac{x}{10} + \frac{1}{5} + 3$$

$$\underline{\underline{y = \frac{x}{10} + \frac{16}{5}}}$$

Tangents and Normals



Find the tangent & normal to the curve

$$y = \frac{x^3}{8} - 3\sqrt{x}$$

at the point when $x = 4$

When $x = 4$, $y = \frac{4^3}{8} - 3\sqrt{4}$

$$= \frac{4 \times 4^2}{8} - 3\sqrt{4}$$

$$= 8 - 6 = 2$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^3}{8} - 3\sqrt{x}\right)$$

$$= \frac{d}{dx}\left(x^3/8 - 3x^{\frac{1}{2}}\right)$$

$$= \frac{3x^2}{8} - \frac{3}{2}x^{-\frac{1}{2}} = \frac{3x^2}{8} - \frac{3}{2\sqrt{x}}$$

at $x = 4$, $\frac{dy}{dx} = \frac{3 \times 4^2}{8} - \frac{3}{2\sqrt{4}} = 6 - \frac{3}{4} = \frac{24}{4} - 3/4 = \frac{21}{4}$

Tangent

$$y - y_0 = m(x - x_0)$$

$$y - 2 = \frac{21}{4}(x - 4)$$

$$y - 2 = \frac{21}{4}x - 21$$

$$\underline{\underline{y = \frac{21}{4}x - 19}}$$

Normal

$$y - y_0 = n(x - x_0)$$

$$y - 2 = \frac{-4}{21}(x - 4)$$

$$y - 2 = \frac{-4}{21}x + \frac{16}{21}$$

$$y = \frac{-4}{21}x + \frac{16}{21} + 2$$

$$\underline{\underline{y = \frac{-4}{21}x + \frac{58}{21}}}$$