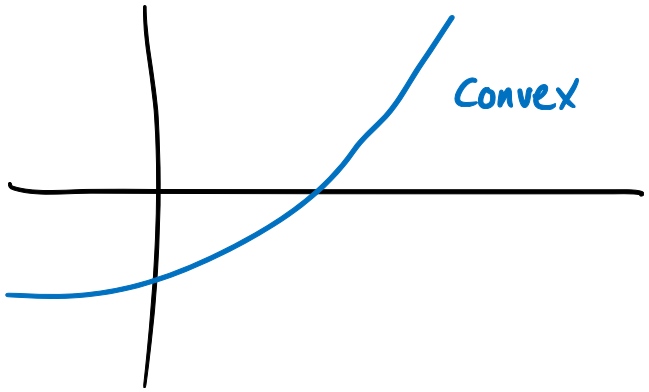
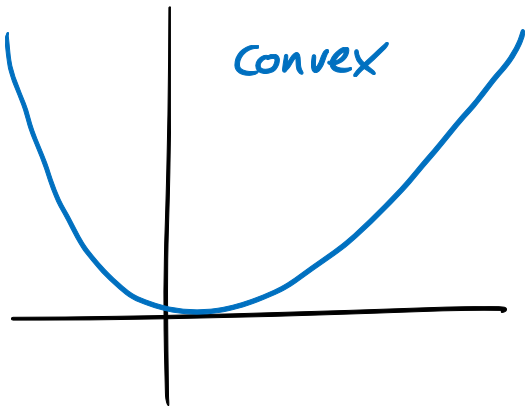
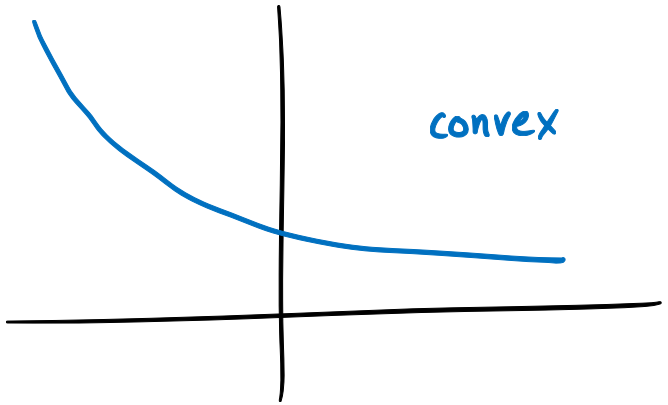
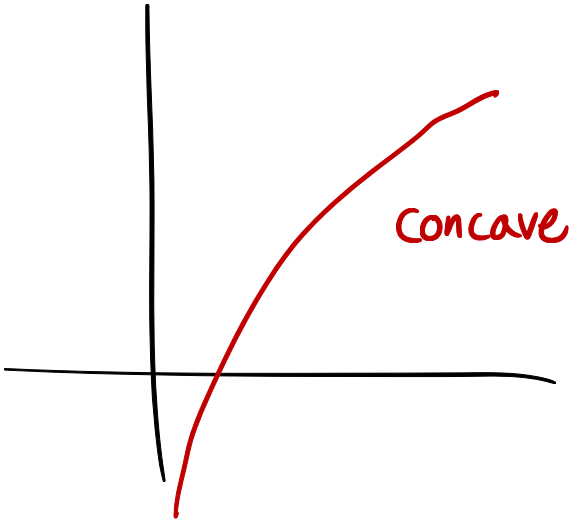
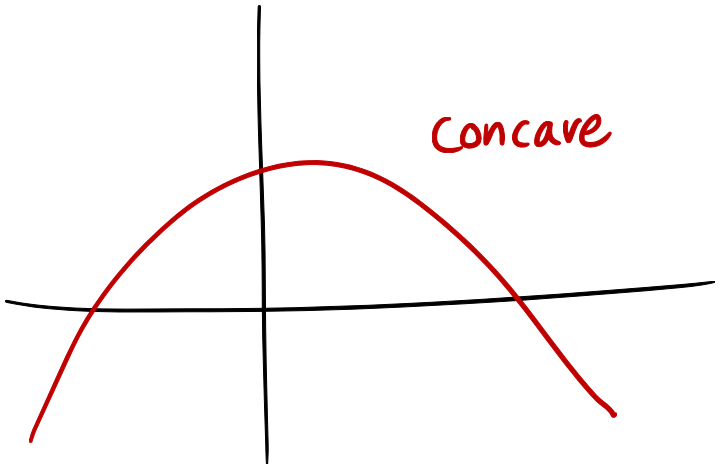
Convexity & Concavity

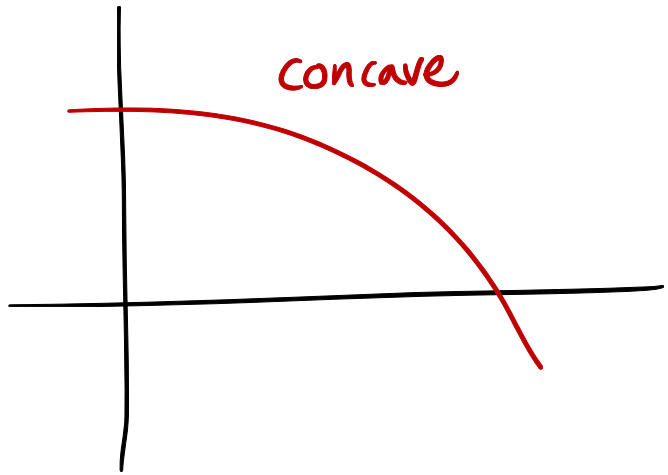




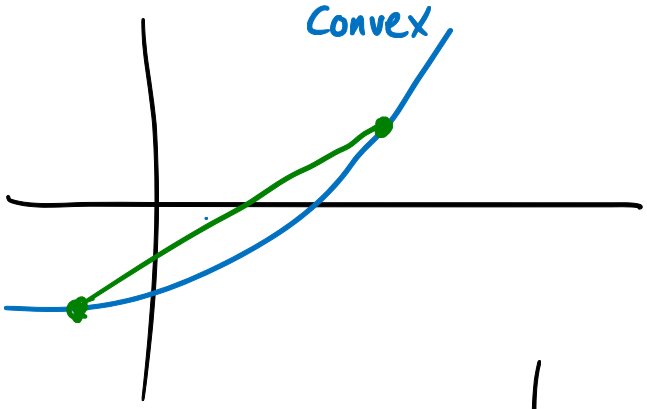


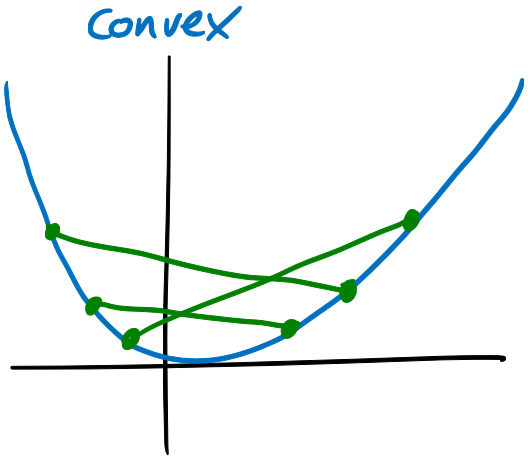


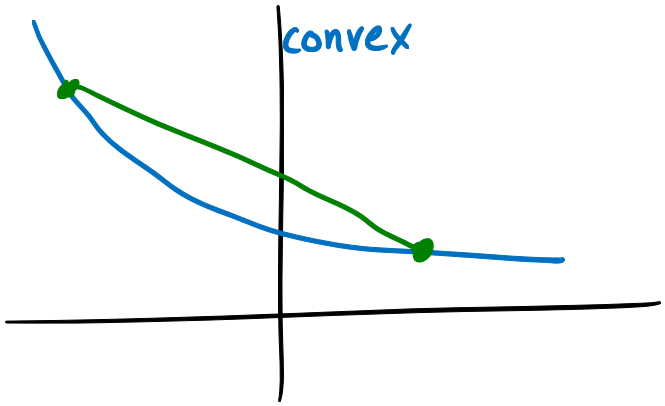


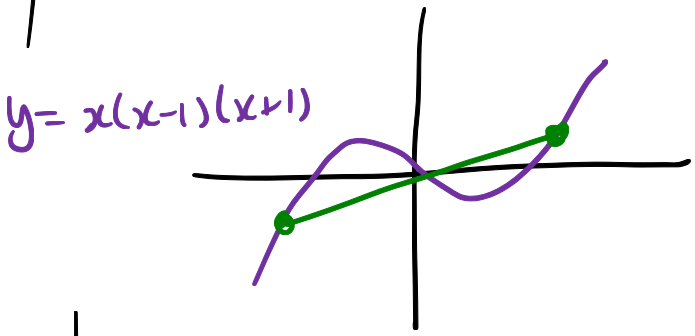


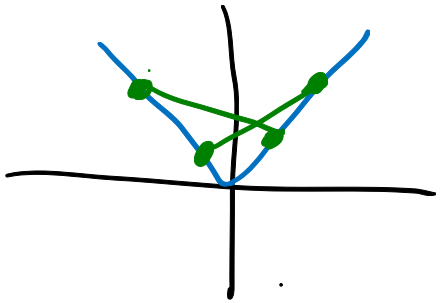
Convexity & Concavity

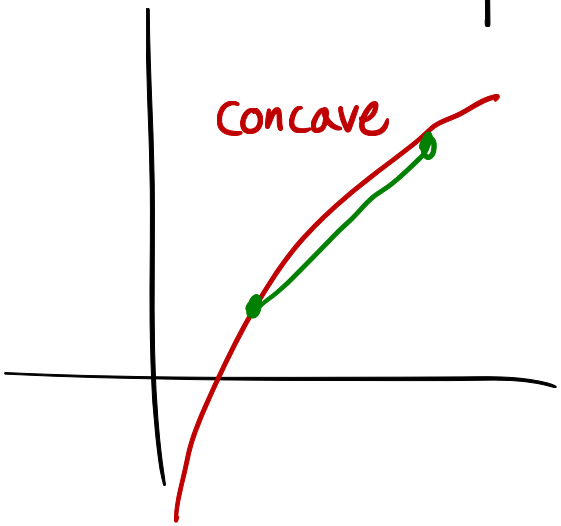


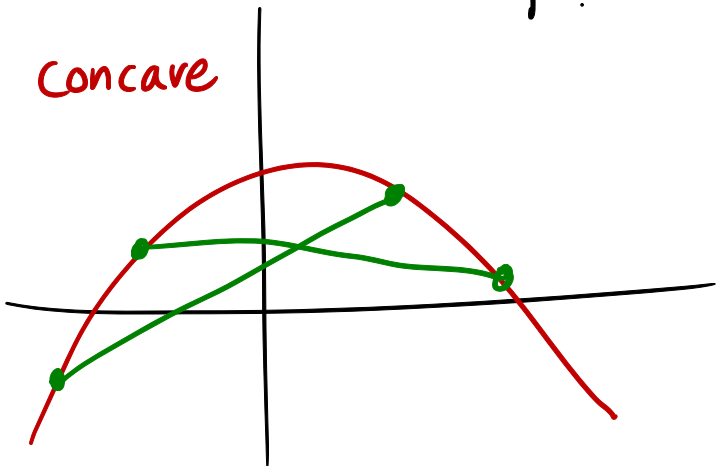


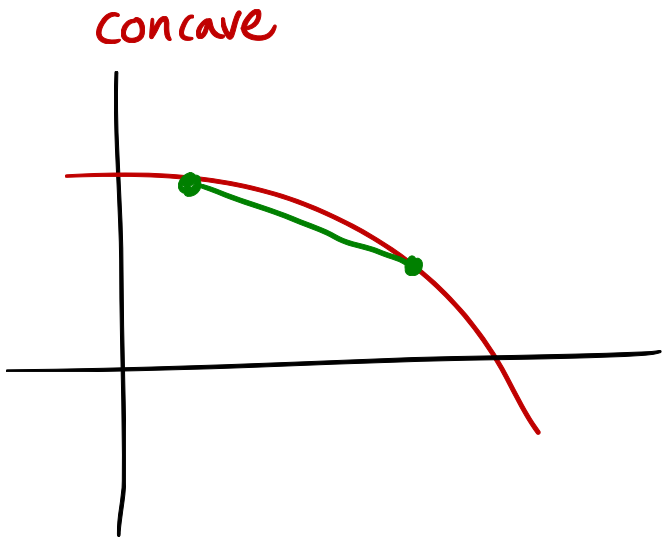




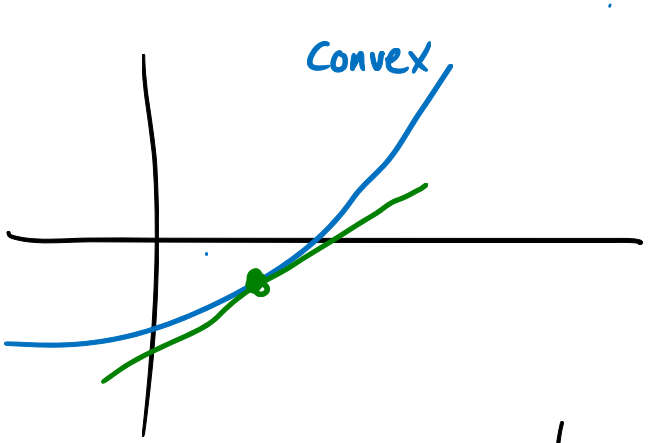


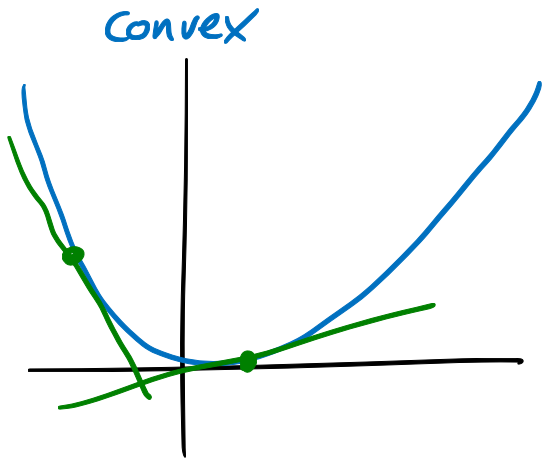


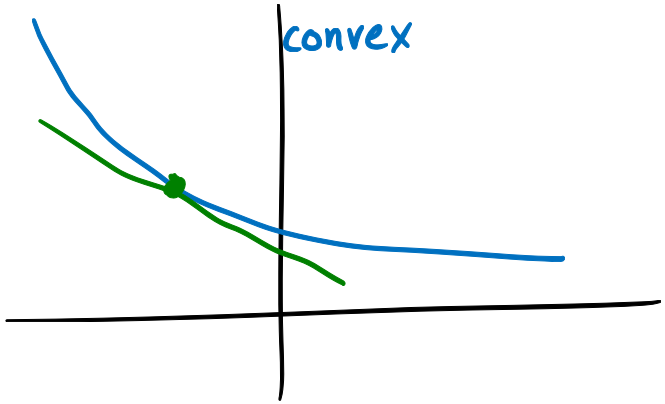


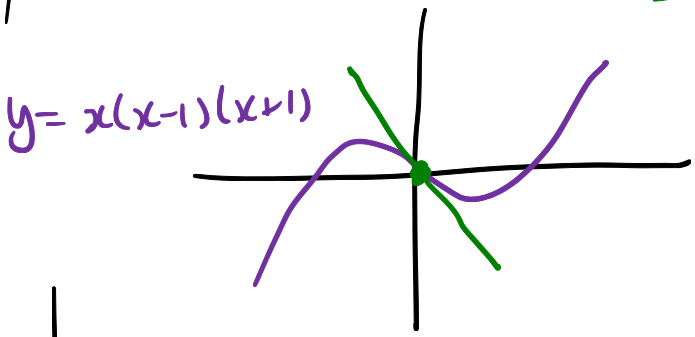


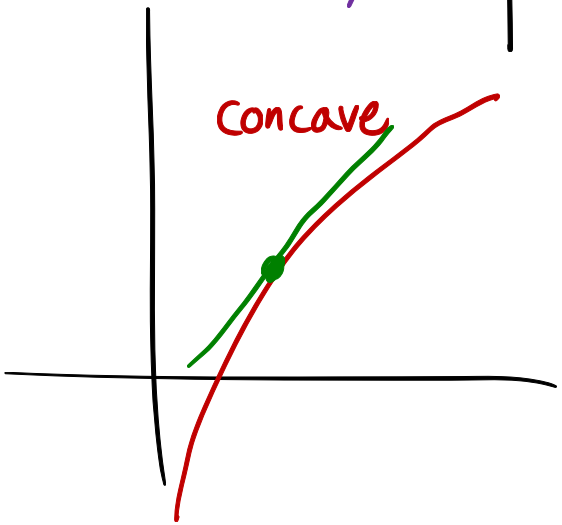
Convexity & Concavity

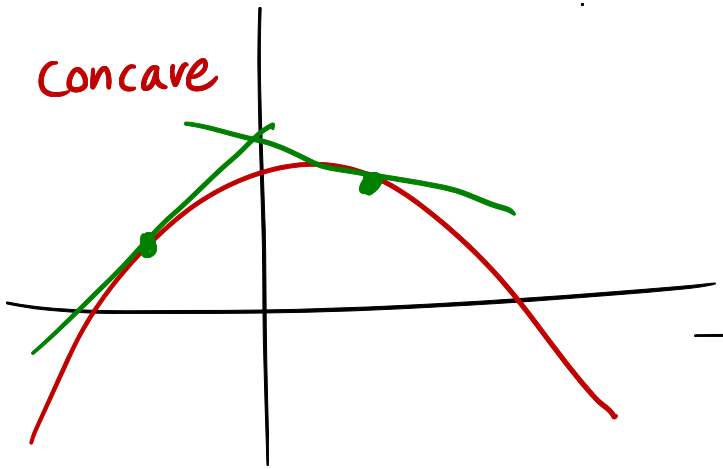


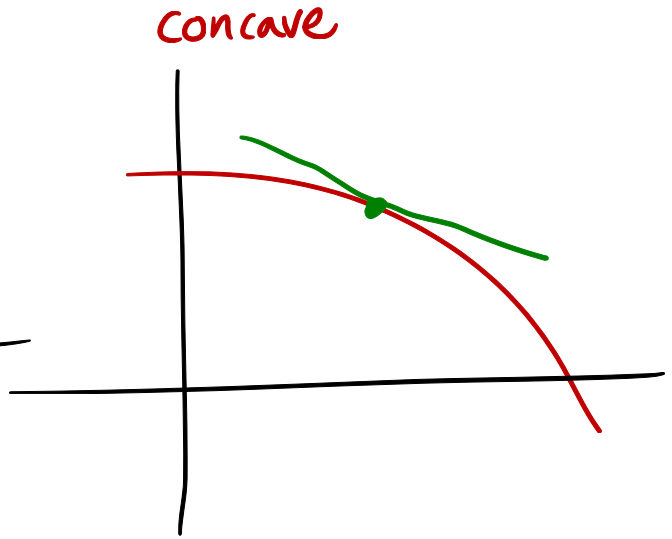




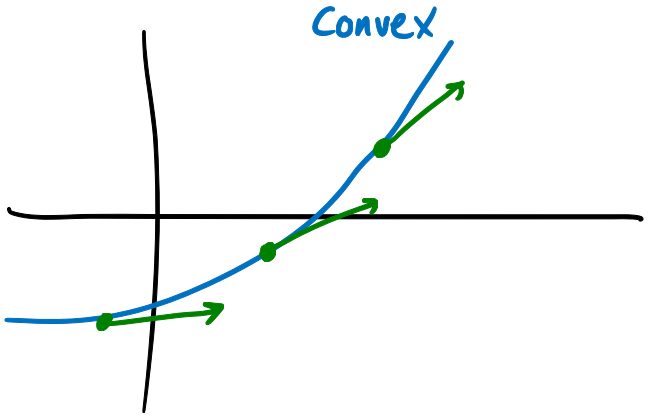


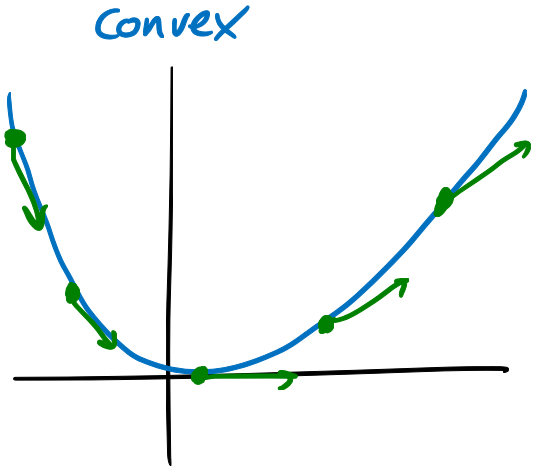


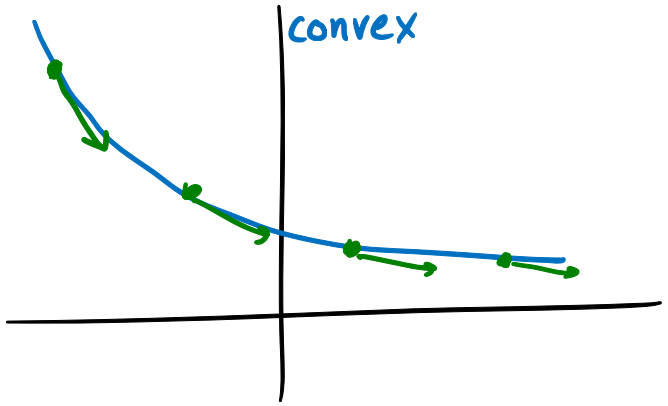




Convexity & Concavity

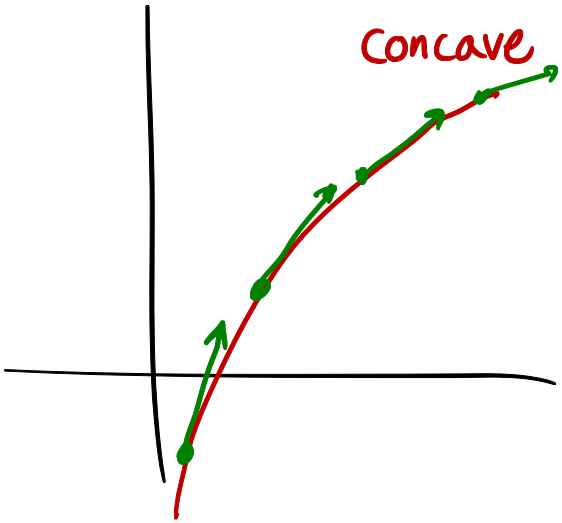


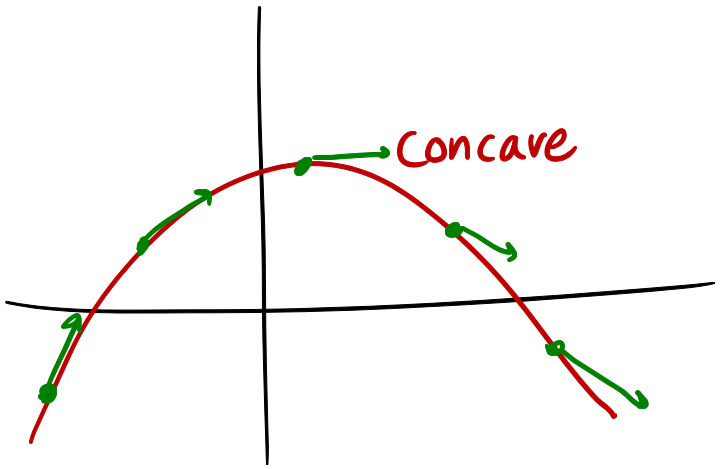


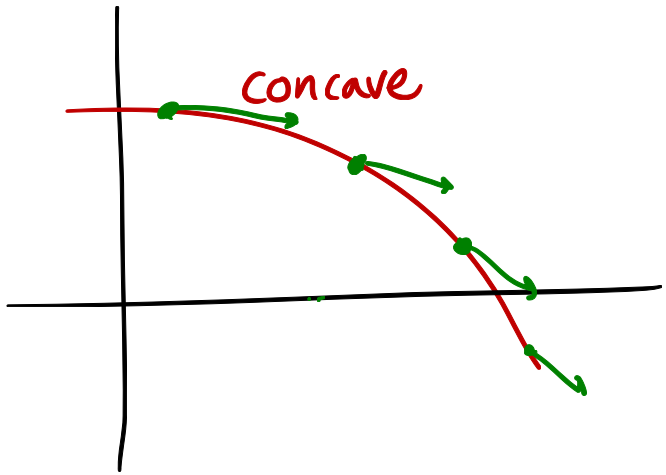


$f'' > 0 \iff (f')' > 0 \Rightarrow f'$ increasing $\Rightarrow f$ convex

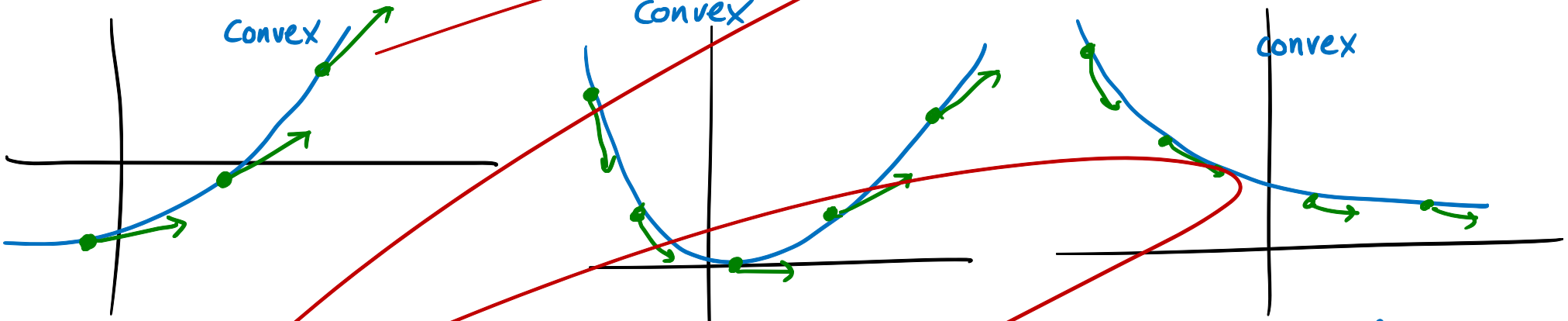
$f'' < 0 \iff (f')' < 0 \Rightarrow f'$ decreasing $\Rightarrow f$ concave





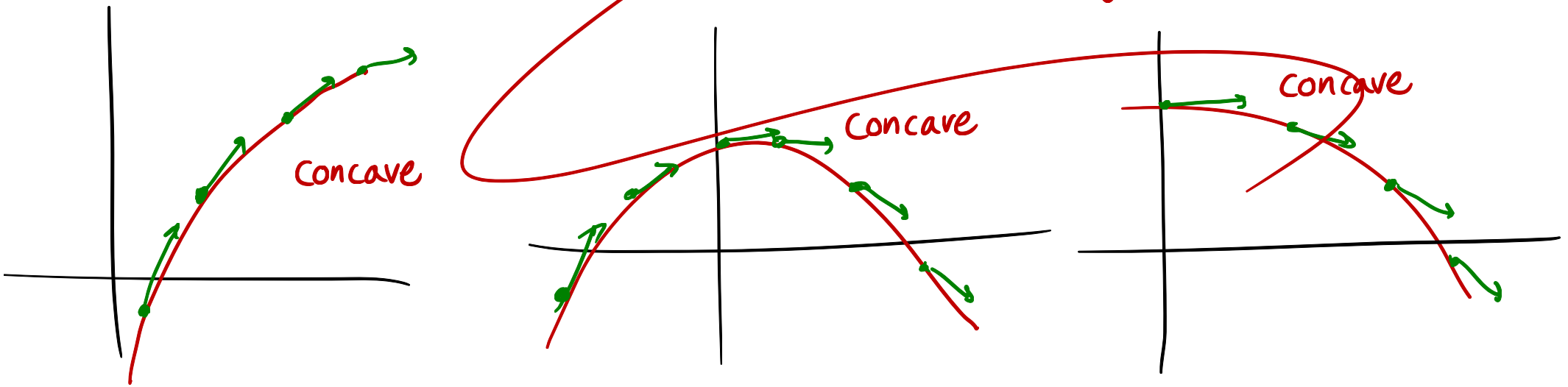


Convexity & Concavity

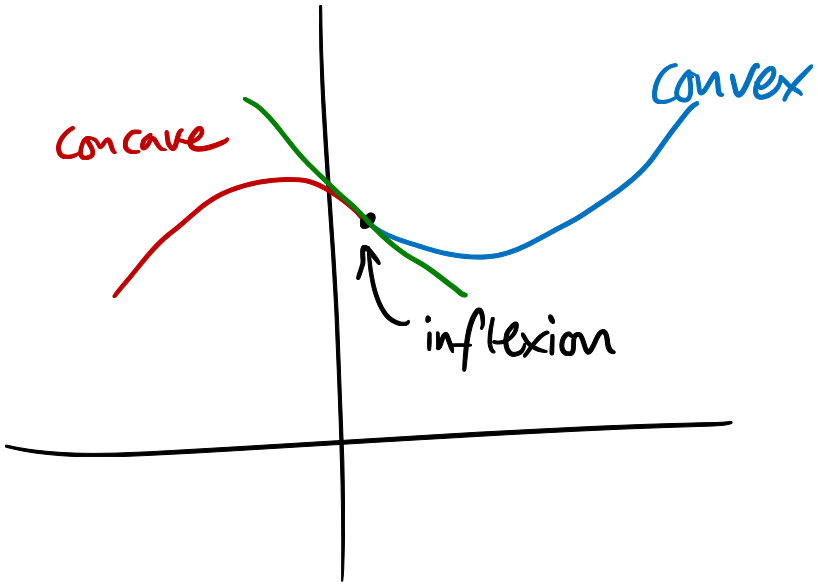


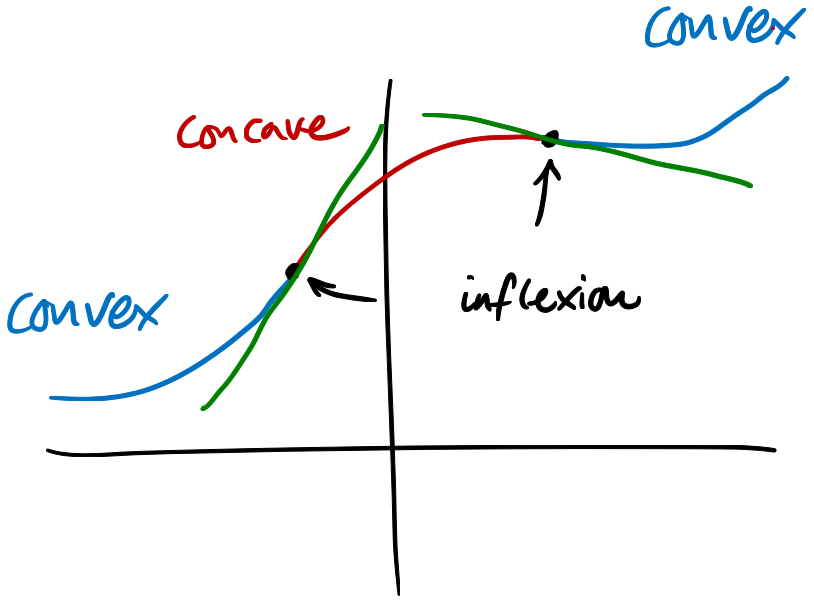
$f'' > 0 \Rightarrow (f')' > 0 \Rightarrow f'$ is increasing $\Rightarrow f$ convex

$f'' < 0 \Rightarrow (f')' < 0 \Rightarrow f'$ decreasing $\Rightarrow f$ is concave



Convexity & Concavity





Convexity & Concavity

$f'' > 0 \Rightarrow$ convex

$f'' < 0 \Rightarrow$ concave

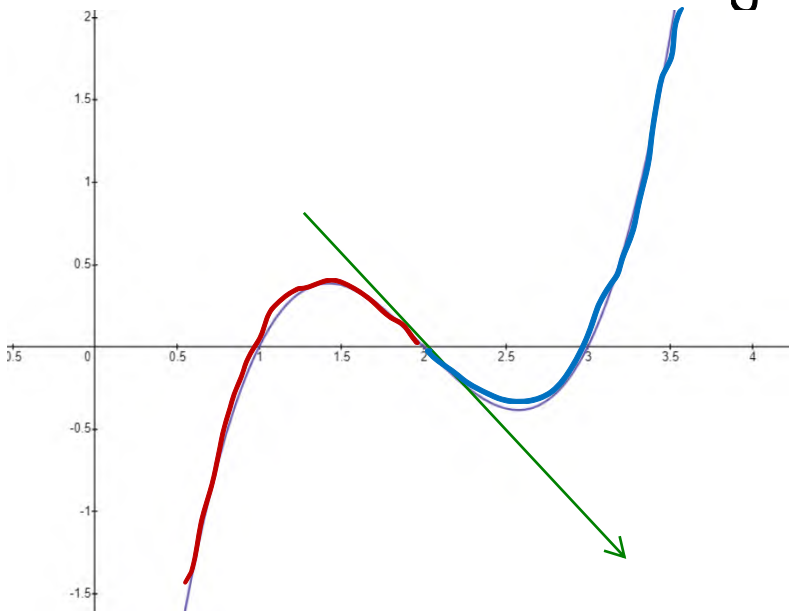
On what regions is the curve $y = x^3 - 6x^2 + 11x - 6$ concave/convex?

$$y' = 3x^2 - 12x + 11$$

$$y'' = 6x - 12$$
$$= 6(x-2)$$

$y'' > 0 \iff x > 2$ convex

$y'' < 0 \iff x < 2$ concave



Convexity & Concavity

$y \geq 0$

intercepts $(0,0)$

Sketch the curve $y = x^2 e^x$

$y' = \frac{d}{dx}(x^2 e^x) = 2xe^x + x^2 e^x$

$= (2x + x^2) e^x$

$= (2+x) x e^x$

Stationary points $x = 0 \Rightarrow y = 0$

$x = -2 \Rightarrow y = 4e^{-2} = \frac{4}{e^2}$

$x < -2 \Rightarrow y' > 0 \Rightarrow y$ increasing

$-2 < x < 0 \Rightarrow y' < 0 \Rightarrow y$ decreasing

$x > 0 \Rightarrow y' > 0 \Rightarrow y$ increasing

$y'' = (2+2x) e^x + (2x + x^2) e^x$

$= (2 + 4x + x^2) e^x$

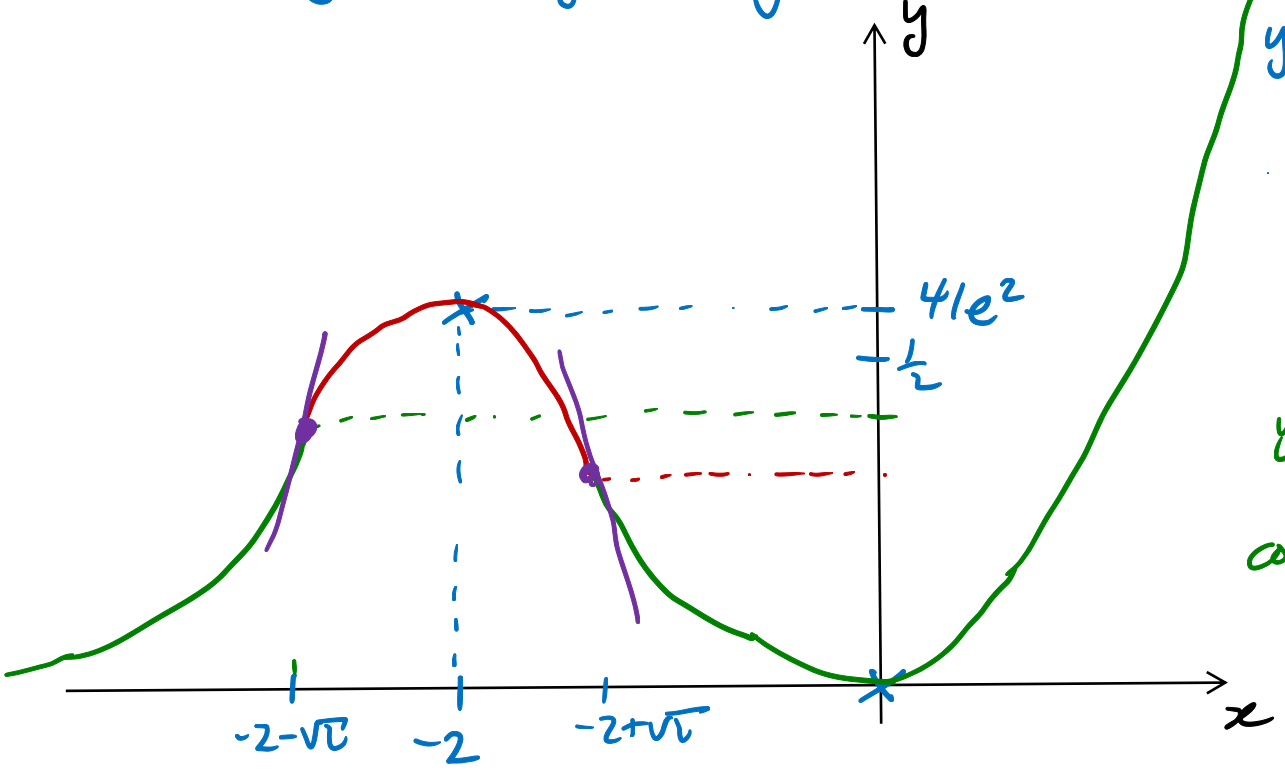
> 0

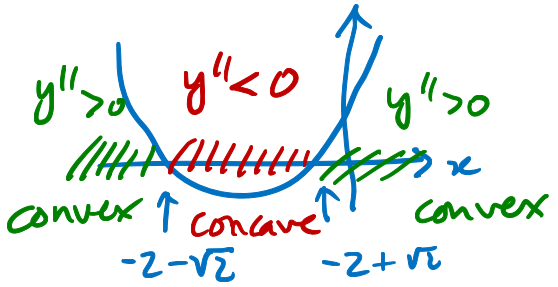
$y'' > 0 \iff 2 + 4x + x^2 > 0$

discriminant $\Delta = 4^2 - 4 \times 1 \times 2 = 8 > 0$

$2 + 4x + x^2 = 0 \iff x = \frac{-4 \pm \sqrt{8}}{2}$

$= -2 \pm \sqrt{2}$





Convexity & Concavity

$y \geq 0$

intercepts $(0,0)$

Sketch the curve $y = x^2 e^x$

$y' = \frac{d}{dx}(x^2 e^x) = 2xe^x + x^2 e^x$

$= (2x + x^2) e^x$

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$x < -2 \Rightarrow y' > 0 \Rightarrow y$ increasing

$-2 < x < 0 \Rightarrow y' < 0 \Rightarrow y$ decreasing

$x > 0 \Rightarrow y' > 0 \Rightarrow y$ increasing

$y'' = (2+2x)e^x + (2x+x^2)e^x$

$= (2 + 4x + x^2) e^x$

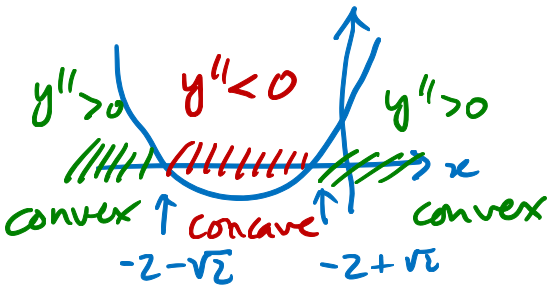
> 0

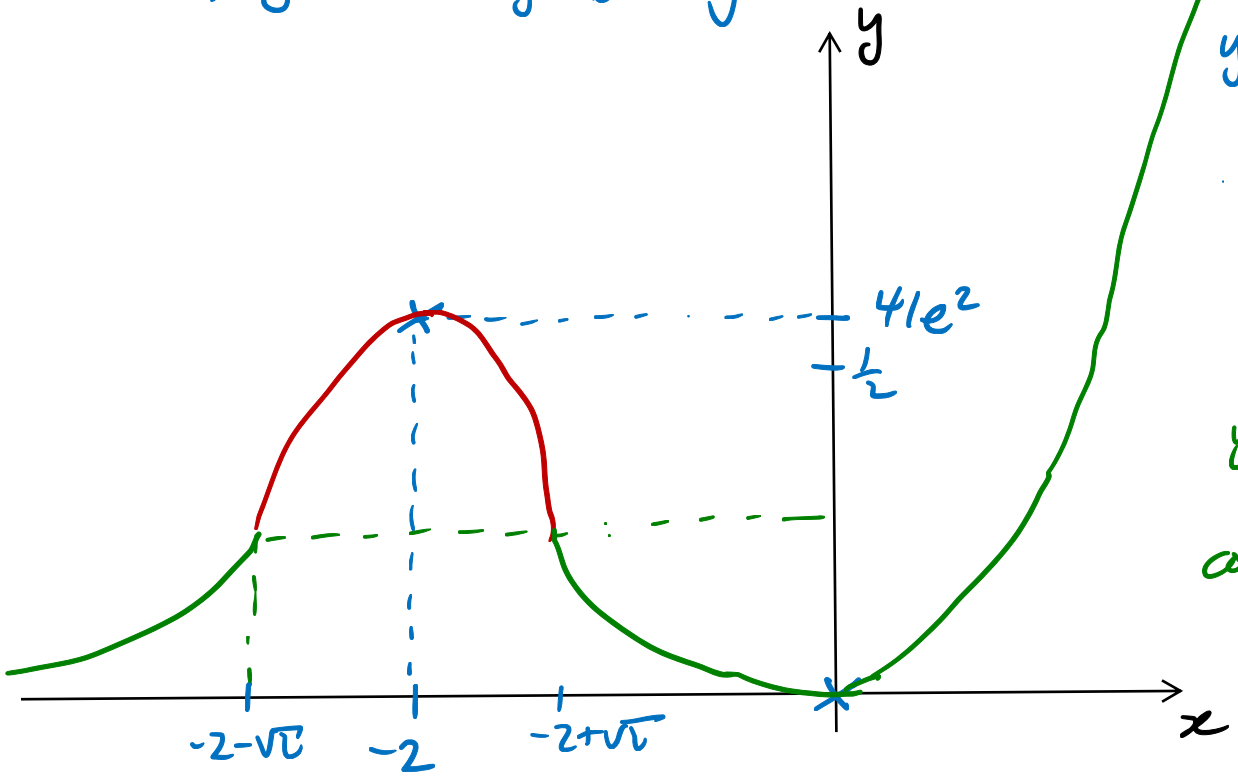
$y'' > 0 \Leftrightarrow 2 + 4x + x^2 > 0$

discriminant $\Delta = 4^2 - 4 \times 1 \times 2 = 8 > 0$

$2 + 4x + x^2 = 0 \Leftrightarrow x = \frac{-4 \pm \sqrt{8}}{2}$

$= -2 \pm \sqrt{2}$





https://www.desmos.com/calculator/lj0y9oab0l