

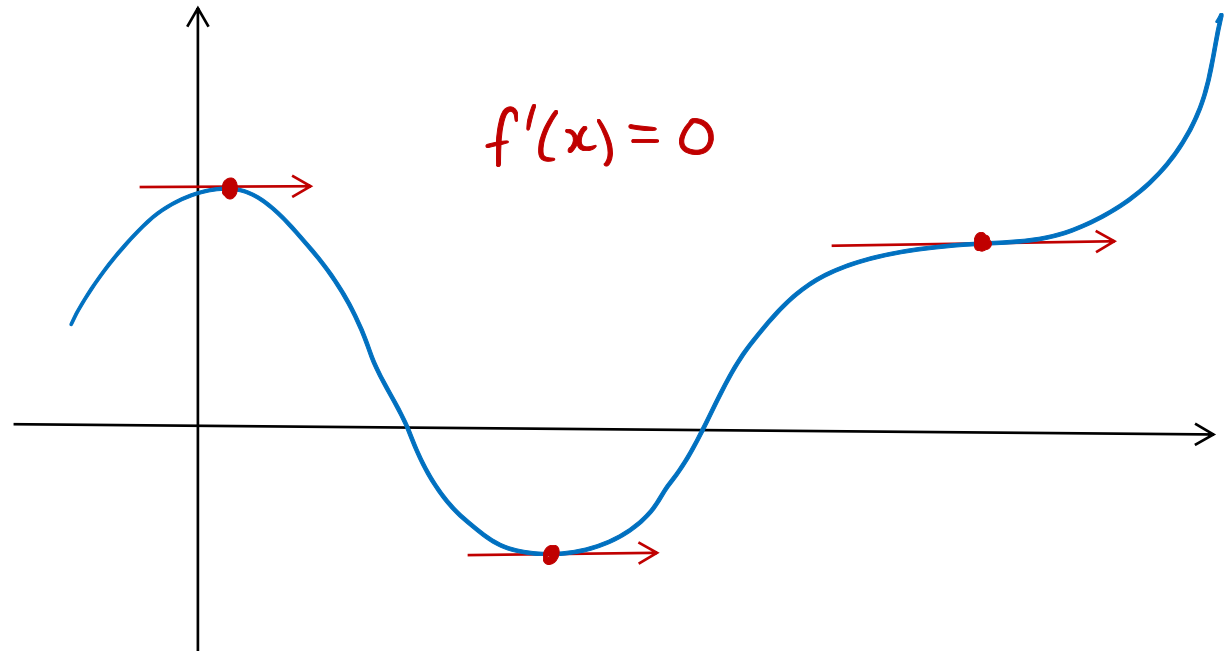
Finding stationary points

Find the stationary points of

$$y = 2x^3 - 3x^2 - 12x + 5$$

$$g(t) = 7 - \frac{3t}{t^2 + 3}$$

$$f(x) = (x^2 - 4x) \exp\left(\frac{x}{2}\right)$$



Finding stationary points

Find the stationary points of $y = 2x^3 - 3x^2 - 12x + 5$

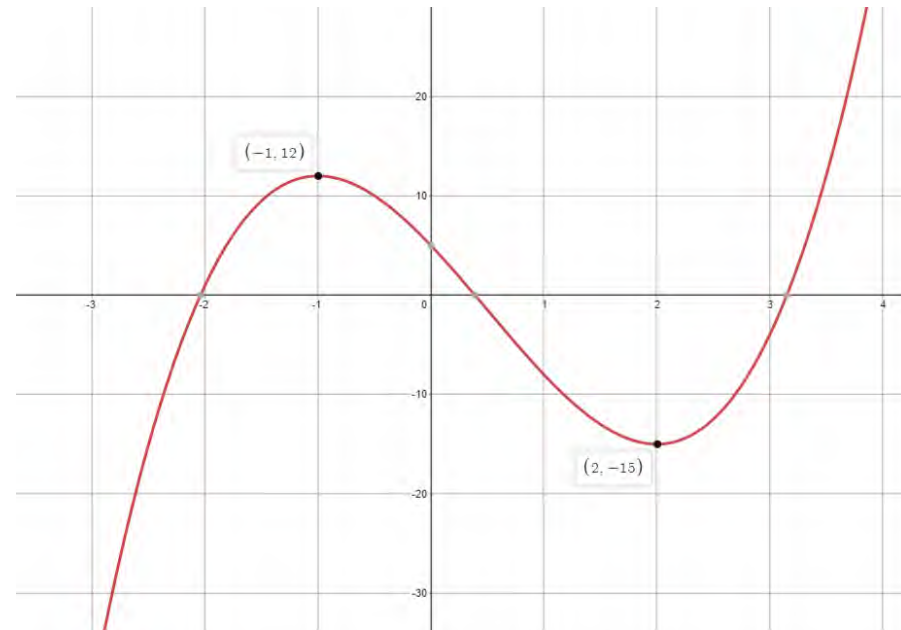
1st order condition: $\frac{dy}{dx} = 0$:

$$\begin{aligned}\frac{dy}{dx} &= 6x^2 - 6x - 12 \\ &= 6(x^2 - x - 2) \\ &= 6(x-2)(x+1)\end{aligned}$$

$$\frac{dy}{dx} = 0 \Leftrightarrow 6(x-2)(x+1) = 0$$

$$\Leftrightarrow \begin{cases} \text{either } x-2=0 \Leftrightarrow x=2 \\ \text{or } x+1=0 \Leftrightarrow x=-1 \end{cases}$$

$$\begin{array}{l} \text{at } x=2, \quad y = 2 \times 2^3 - 3 \times 2^2 - 12 \times 2 + 5 \\ \quad \quad \quad = 16 - 12 - 24 + 5 = -15 \\ \quad \quad \quad \underline{\underline{(2, -15)}} \end{array} \quad \Bigg| \quad \begin{array}{l} \text{at } x=-1 \\ y = 2 \times (-1)^3 - 3(-1)^2 - 12(-1) + 5 \\ \quad \quad \quad = -2 - 3 + 12 + 5 \\ \quad \quad \quad = 12, \quad \underline{\underline{(-1, 12)}} \end{array}$$



Finding stationary points

Find the stationary points of $g(t) = 7 - \frac{3t}{t^2+3}$

$$g'(t) = \frac{d}{dt} \left(\frac{-3t}{t^2+3} \right) \xrightarrow[\text{rule}]{\text{quotient}} \frac{(-3t)(2t) - (-3)(t^2+3)}{(t^2+3)^2}$$

$$= \frac{-6t^2 + 3t^2 + 9}{(t^2+3)^2} = \frac{-3t^2+9}{(t^2+3)^2}$$

f.o.c. $g'(t) = 0 \Leftrightarrow \frac{-3t^2+9}{(t^2+3)^2} = 0$

$\Leftrightarrow -3t^2+9=0$

$\Leftrightarrow 9=3t^2$

$\Leftrightarrow 3=t^2$

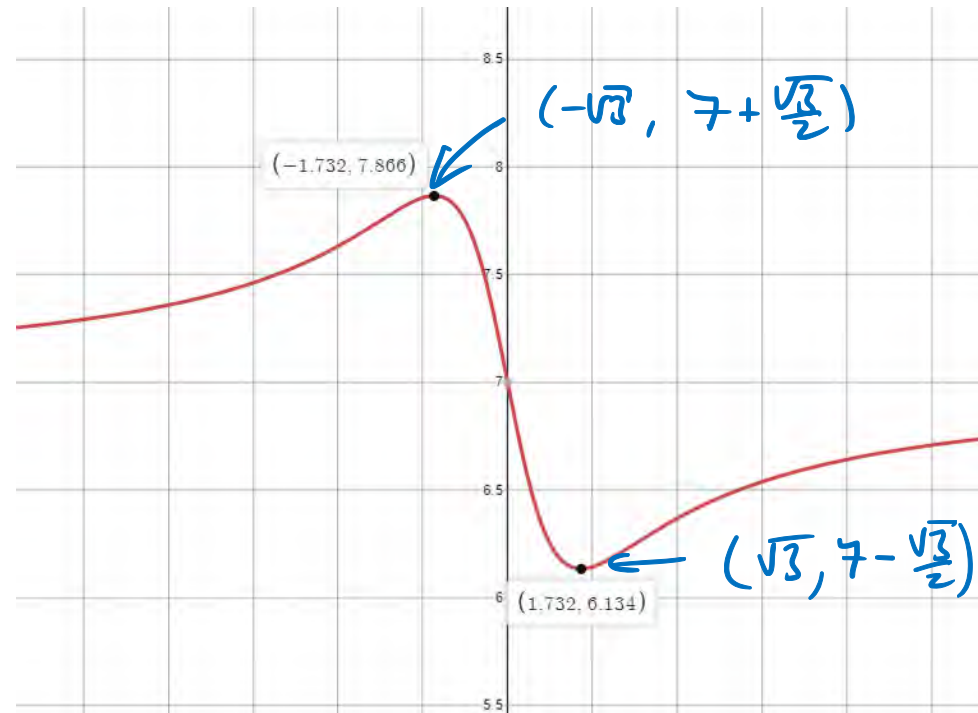
$\Leftrightarrow t = \pm\sqrt{3}$

$$g(\sqrt{3}) = 7 - \frac{3\sqrt{3}}{3+3}$$

$$= 7 - \frac{\sqrt{3}}{2}$$

$$g(-\sqrt{3}) = 7 - \frac{3(-\sqrt{3})}{3+3}$$

$$= 7 + \frac{\sqrt{3}}{2}$$



Finding stationary points

Find the stationary points of $f(x) = (x^2 - 4x) \exp\left(\frac{x}{2}\right)$

$$f'(x) = (2x-4)e^{x/2} + (x^2-4x)\frac{1}{2}e^{x/2}$$

$$= \frac{1}{2}e^{x/2}(4x-8 + x^2-4x)$$

$$= \frac{1}{2}e^{x/2}(x^2-8)$$

F.O.C. $f'(x) = 0 \Leftrightarrow \frac{1}{2}e^{x/2}(x^2-8) = 0$

$\neq 0$

$$\Leftrightarrow x^2 - 8 = 0$$

$$\Leftrightarrow x^2 = 8$$

$$\Leftrightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

$$f(2\sqrt{2}) = (8 - 8\sqrt{2}) \exp(\sqrt{2})$$

$$f(-2\sqrt{2}) = (8 + 8\sqrt{2}) \exp(-\sqrt{2})$$

