Classifying stationary points

Find and classify the stationary points of $y = 5x^3(x-4)$

Sketch the graph of $\pi(t) = \frac{4t}{t^2+4}$, find and classify its stationary points

Find and classify the stationary points of $f(a) = a^4$

Classifying stationary points

at stationary point

$f'' > 0 \Rightarrow$ local min

$f'' < 0 \Rightarrow$ local max

Find and classify the stationary points of $y = 5x^3(x-4) = 5x^4 - 20x^3$

F.O.C $y' = 0$. $y' = 20x^3 - 60x^2 = 20x^2(x-3)$

so $x = 0$ or $x = 3$

$y'' = 60x^2 - 120x = 60x(x-2)$.

at $x=3$, $y'' = 60 \times 3 \times (3-2) = 180 > 0$ so local min at $x = 3$

$y = 5 \times 3^3(3-4) = 5 \times 27 \times (-1) = -135$

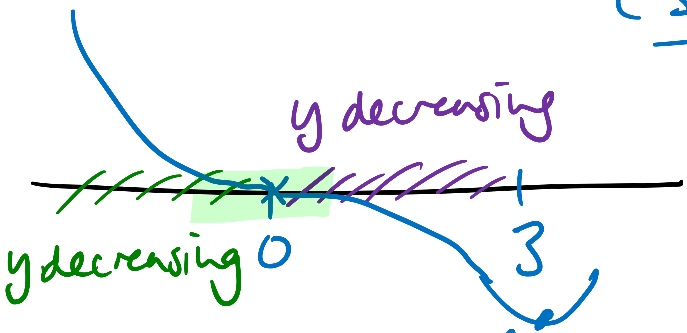
local min at $(3, -135)$

at $x=0$, $y = 5 \times 0^3(0-4) = 0$

$y'' = 60 \times 0 \times (0-2) = 0$

$x < 0$, $y' = 20\underbrace{x^2}_{>0}\underbrace{(x-3)}_{<0} < 0$

$0 < x < 3$, $y' = 20\underbrace{x^2}_{>0}\underbrace{(x-3)}_{<0}$



$(0,0)$ point of inflexion

Classifying stationary points

Sketch the graph of $\pi(t) = \dfrac{4t}{t^2+4}$, find and classify its stationary points

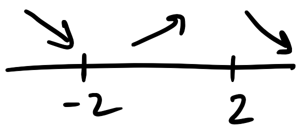
$$\pi'(t) = \frac{4(t^2+4) - 4t(2t)}{(t^2+4)^2}$$

$$= \frac{-4t^2+16}{(t^2+4)^2}$$

$$= \frac{4(4-t^2)}{(t^2+4)^2}$$

$$\pi'(t) \geq 0 \iff 4(4-t^2) \geqslant 0$$

$$\iff 4-t^2 \geqslant 0 \iff (2-t)(2+t) \geqslant 0$$



	$2-t$	$2+t$	π'	π
$t<-2$	+	−	−	↘
$-2<t<2$	+	+	+	↗
$t>2$	−	+	−	↘

Stationary points $\pi' = 0$

when $4-t^2=0$

$t^2=4$

$t=\pm 2$

π has local min at $t=-2$

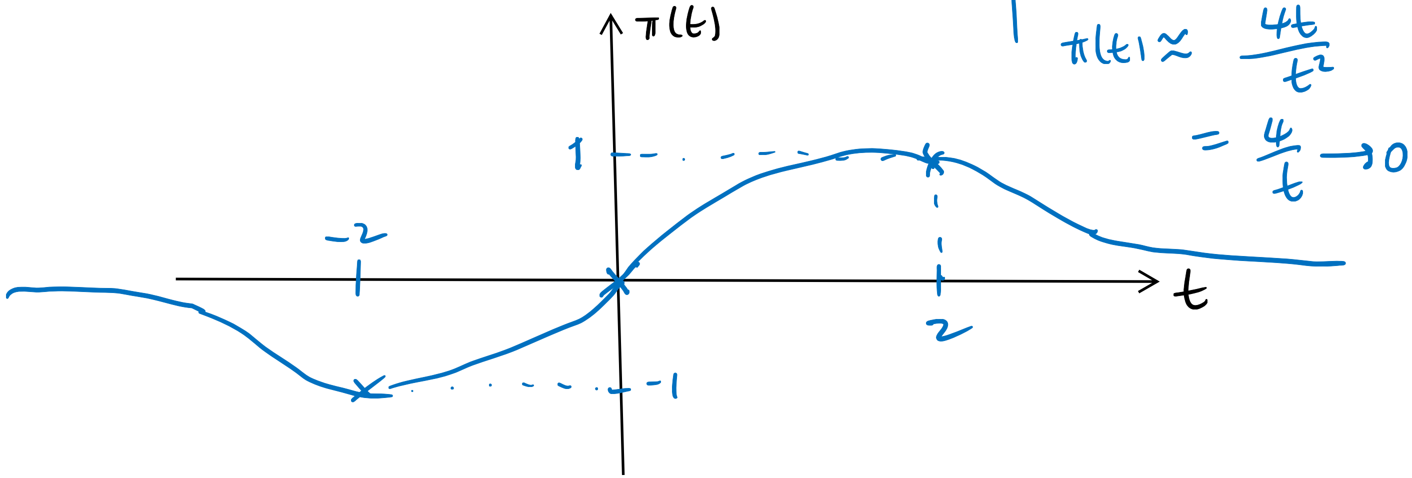
π has local max at $t=2$

For $|t|$ large

$t^2+4 \simeq t^2$

$\pi(t) \simeq \dfrac{4t}{t^2}$

$= \dfrac{4}{t} \to 0$



$$\pi(2) = \frac{8}{8} = 1$$

$$\pi(-2) = -1$$

$$\left(\pi''(t) = \frac{8t(t^2-12)}{(t^2+3)^3}\right)$$

Classifying stationary points

Find and classify the stationary points of $f(a) = a^4$

F.O.C. $\frac{df}{da} = 0$: $\frac{df}{da} = 4a^3$ $\frac{df}{da} = 0 \iff a = 0$

So only stationary point, $\underline{a=0}$.

$\frac{d^2f}{da^2} = 12a^2$, at $a=0$, $\frac{d^2f}{da^2} = 0$

$f(a) = a^4 \geq 0 = f(0)$ 0 is a minimum.