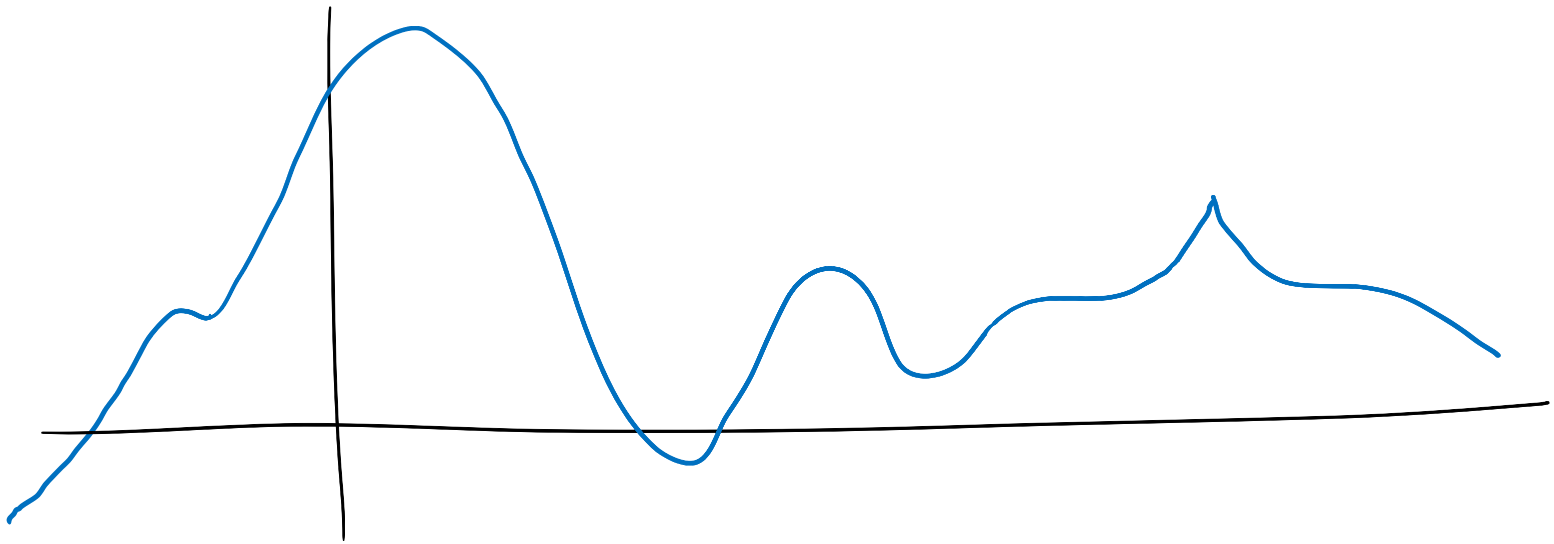
Optimization: Stationary points, Turning points & Critical points



Find the largest & smallest values of $y = x^3 - 7x^2 - 5x - 2$ over the for x in the range $-2 \leq x \leq 8$

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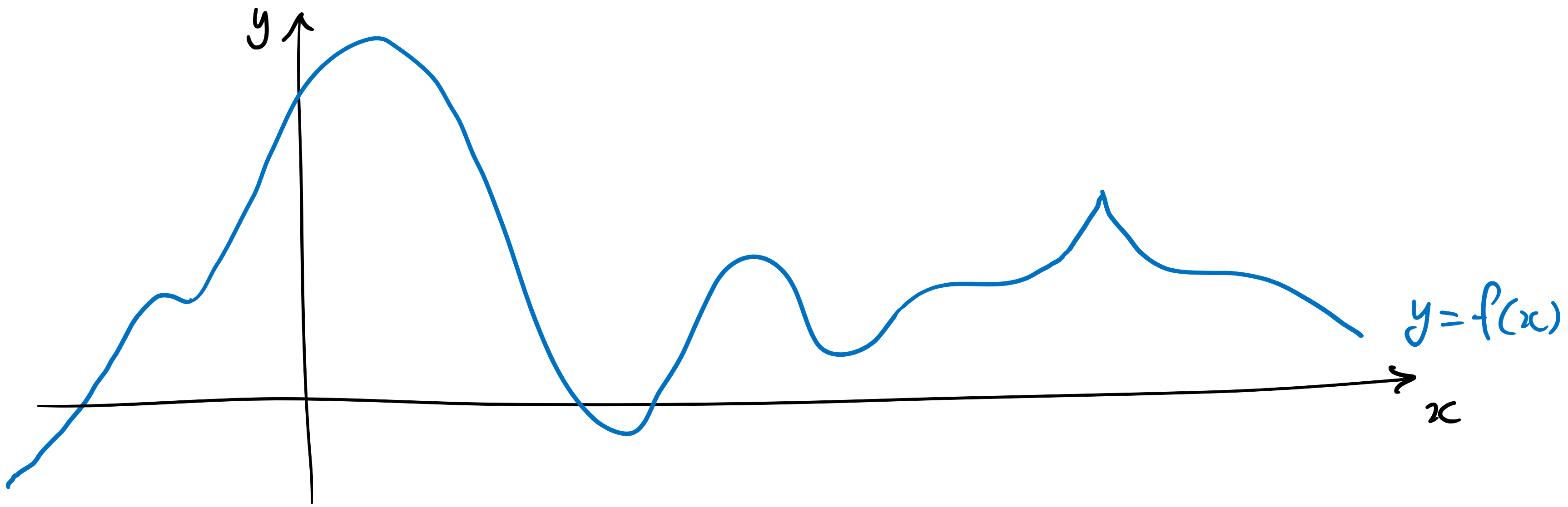

Local extrema

- x_0 is a local maximum if $f(x_0) \geq f(x)$ for all x nearby to x_0
- x_0 is a local minimum if $f(x_0) \leq f(x)$ for all x nearby to x_0

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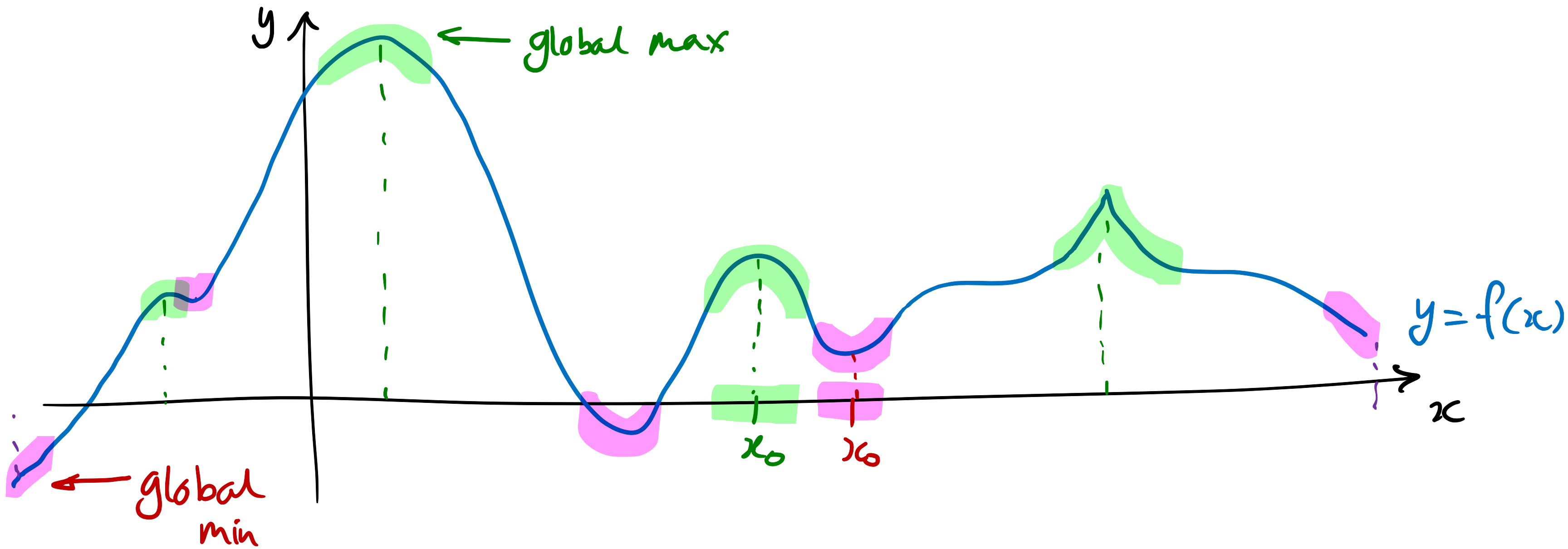

Local extrema

- f has a **local maximum** at x_0 if $f(x_0) \geq f(x)$ for all x nearby to x_0
- f has a **local minimum** at x_0 if $f(x_0) \leq f(x)$ for all x nearby to x_0

Global extrema

- f has a **global maximum** at x_0 if $f(x_0) \geq f(x)$ for all x
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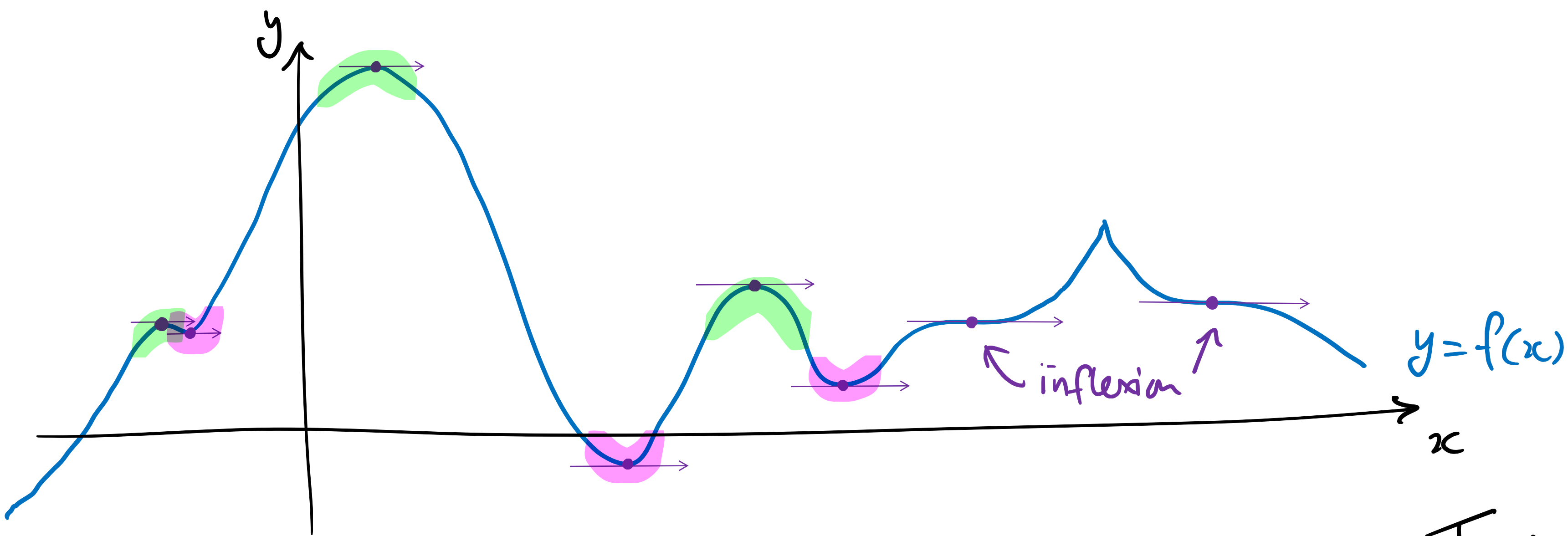
Optimization: Stationary points, Turning points & Critical points

Stationary point $f'(x) = 0$

$$\frac{dy}{dx} = 0$$



Local extrema

- f has a local maximum at x_0 if $f(x_0) \geqslant f(x)$ for all x nearby to x_0
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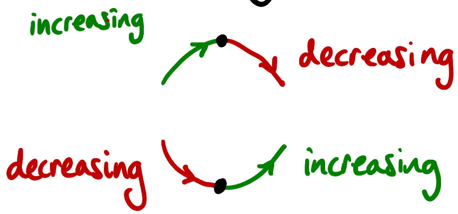
Turning points



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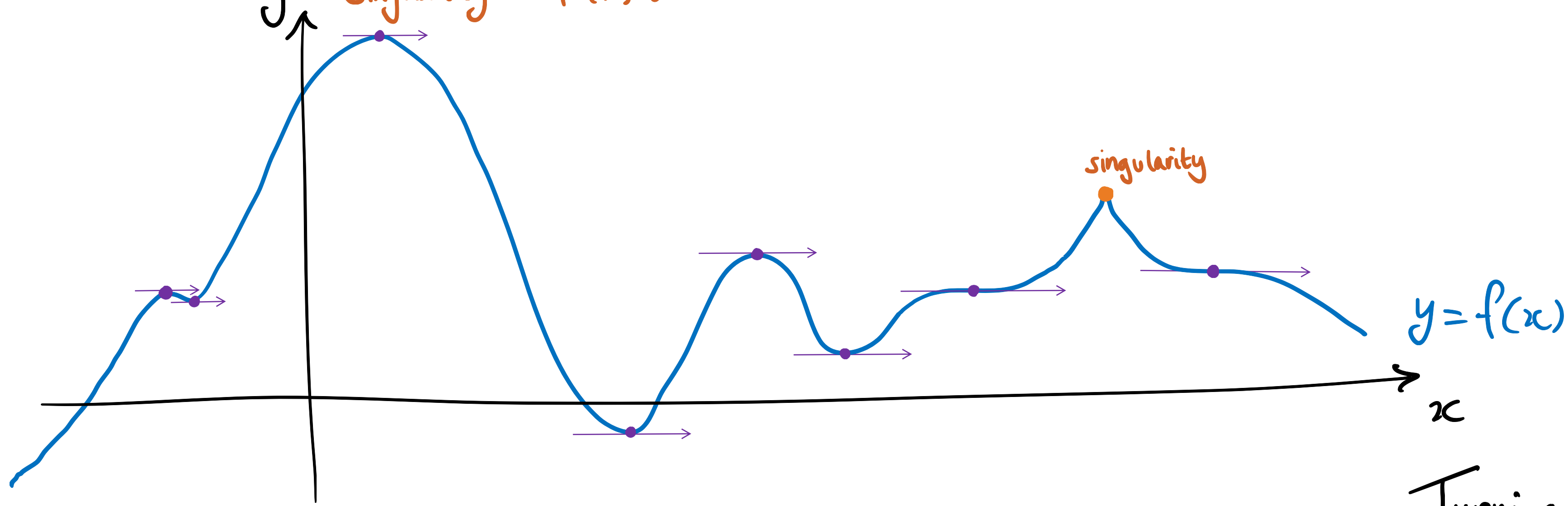
Optimization: Stationary points, Turning points & Critical points

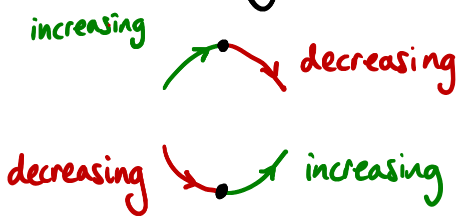
Stationary point $f'(x) = 0$

$$\frac{dy}{dx} = 0$$

Singularity $\quad f'(x)$ doesn't exist



Turning points



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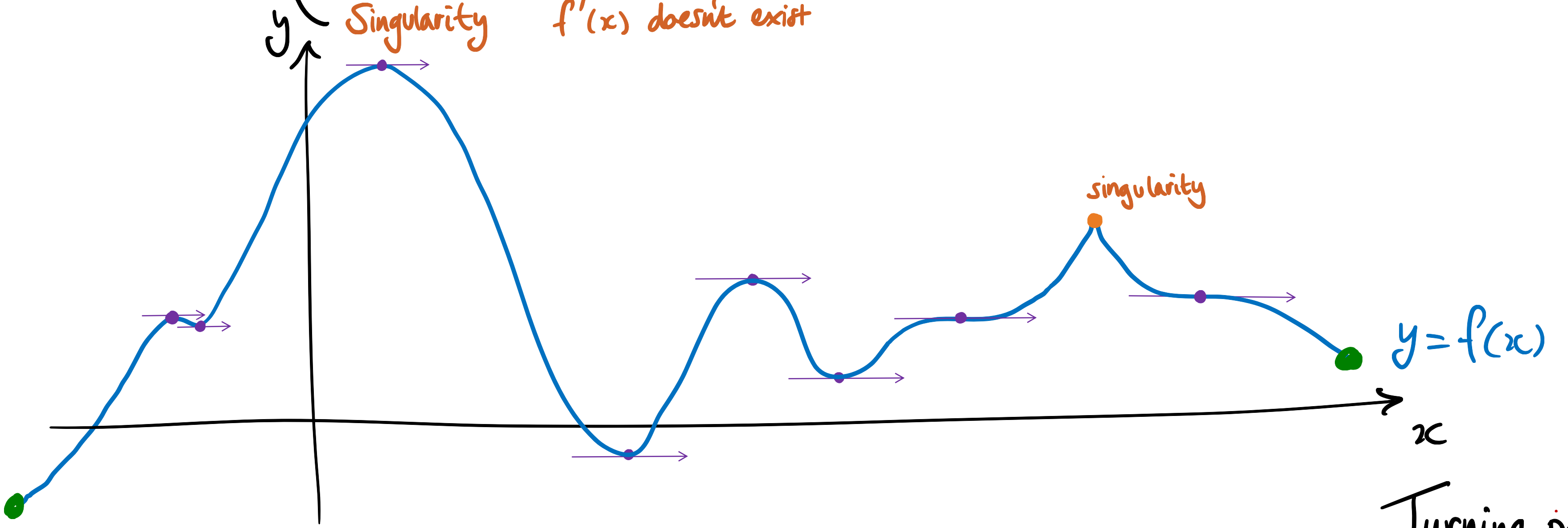
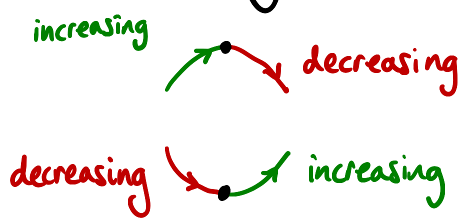
Optimization: Stationary points, Turning points & Critical points

Critical points
$$\begin{cases} \text{Stationary point} & f'(x) = 0 \\ & \dfrac{dy}{dx} = 0 \\ \text{Singularity} & f'(x) \text{ doesn't exist} \end{cases}$$



Turning points



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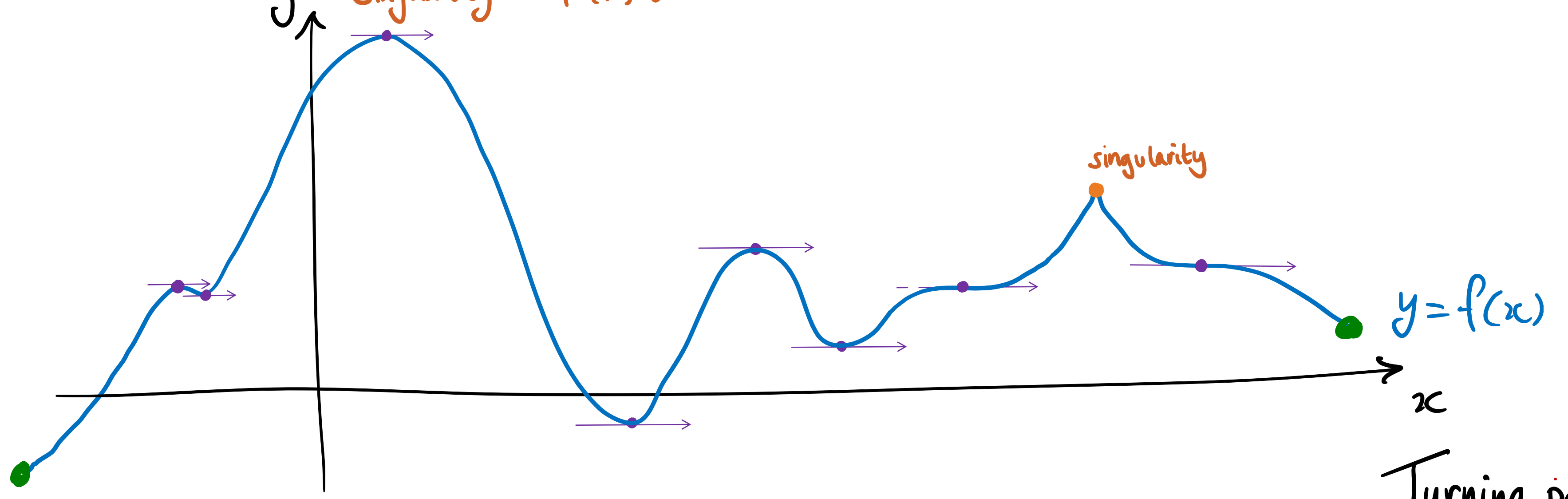
Optimization: Stationary points, Turning points & Critical points

Critical points:
- Stationary point $f'(x)=0$, $\frac{dy}{dx}=0$
- Singularity $f'(x)$ doesn't exist

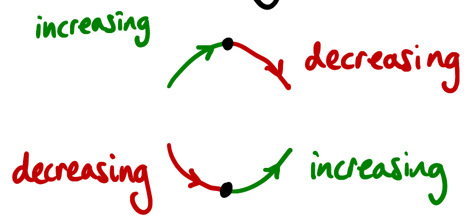
2nd Derivative Test

$f''(x)<0 \Rightarrow$ max

$f''(x)>0 \Rightarrow$ min



Turning points



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Optimization: Stationary points, Turning points & Critical points

Find the largest & smallest values of $y = x^3 - 7x^2 - 5x - 2$ over the for x in the range $-2 \le x \le 8$

Stationary points ($y' = 0$)

$$y' = 3x^2 - 14x - 5 = (x-5)(3x+1)$$

$$y'' = 6x - 14$$

~~singularities~~

endpoints

$$y(-2) = (-2)^3 - 7(-2)^2 - 5(-2) - 2 = -8 - 7\times 4 + 10 - 2 = -28$$

- $x = 5$

$$y(5) = 5^3 - 7\times 5^2 - 5\times 5 - 2 = 5\times 5^2 - 7\times 5^2 - 5^2 - 2 = -3\times 5^2 - 2 = -75 - 2 = \underline{-77}$$

$$y''(5) = 6\times 5 - 14 = 30 - 14 = 16 > 0$$

Local min

$$y(8) = 8^3 - 7\times 8^2 - 5\times 8 - 2 = 8\times 64 - 7\times 64 - 40 - 2 = 64 - 40 - 2 = 22$$

- $x = -1/3$

$$y\left(-\tfrac{1}{3}\right) = \left(-\tfrac{1}{3}\right)^3 - 7\times\left(-\tfrac{1}{3}\right)^2 - 5\left(-\tfrac{1}{3}\right) - 2 = -\frac{1}{27} - \frac{7}{9} + \frac{5}{3} - 2 = \frac{-1 - 21 + 45 - 54}{27} = \underline{-\frac{31}{27}}$$

$$y''\left(-\tfrac{1}{3}\right) = 6\times\left(-\tfrac{1}{3}\right) - 14 = -2 - 14 = -16 < 0$$

Local max