

Arithmetic Progression

4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, ...

17, 15, 13, 11, 9, 7, 5, 3, 1, -1, -3, -5, -7, -9, ...

What are the next 3 terms of the arithmetic sequence that starts 47, 33, 19?

Which of these are arithmetic progressions?

① 9, 16, 23, 29, 35, 40, 45, 49, 53, 56, 59, 61, 63, ...

② 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, ...

③ 111, 106, 101, 96, 91, 86, 81, 76, 71, 66, 61, ...

④ (a_n) given by $a_n = 3n + 2$

⑤ $x(0), x(1), x(2), x(3), \dots$ where $x(k) - x(k-1) = x(k+1) - x(k)$ for $k \geq 1$

What is the 100th term of the arithmetic progression 4, 7, 10, ...?

Suppose that the 3rd and 15th terms of an arithmetic progression sum to -34 and the 6th and 20th terms sum to -58. What is the n^{th} term?

Arithmetic Progression

linear

4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, ...

+3 +3 +3

+3

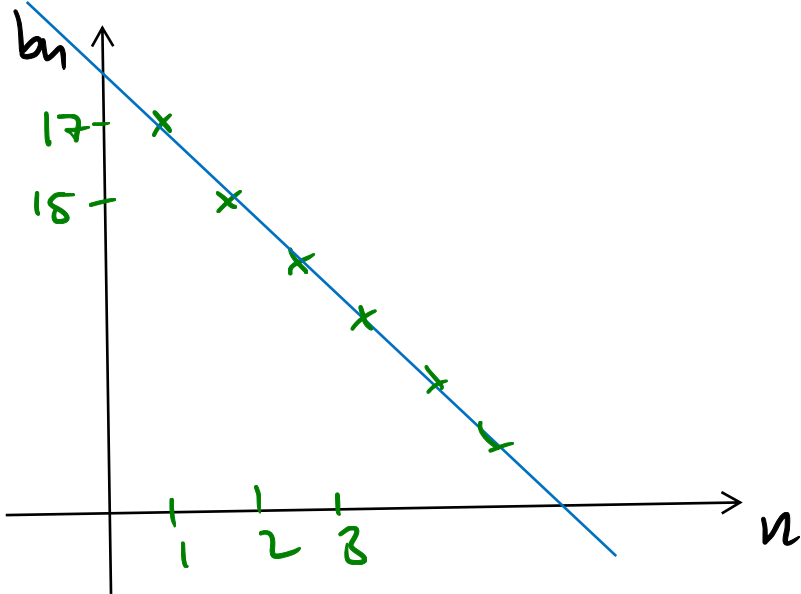
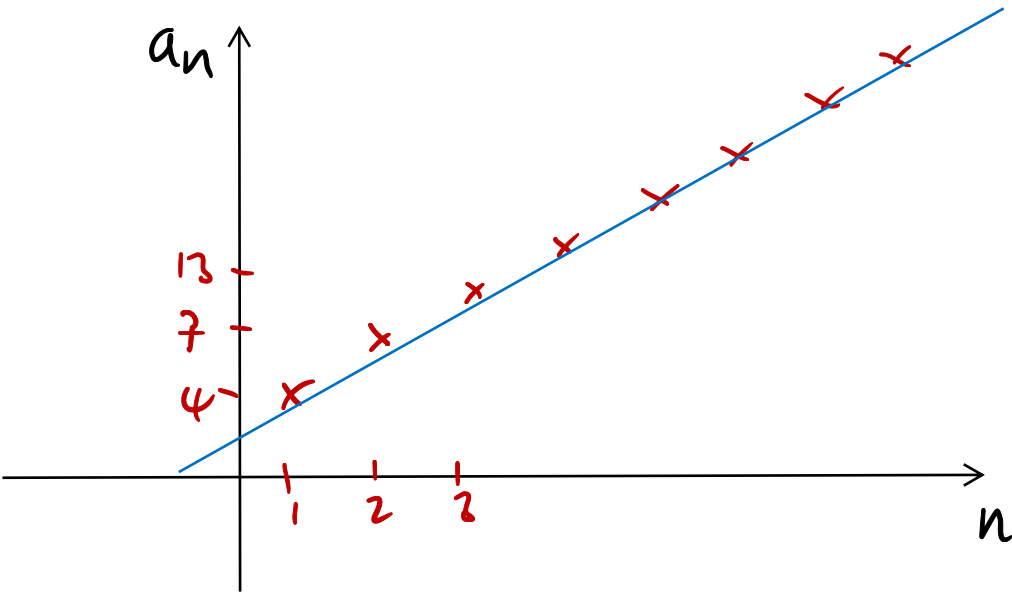
$d=3$ common difference

17, 15, 13, 11, 9, 7, 5, 3, 1, -1, -3, -5, -7, -9, ...

+(-2) +(-2)

+(-2)

$d=-2$



Arithmetic Progression

What are the next 3 terms of the arithmetic sequence that starts 47, 33, 19 ?

$$\begin{array}{cccccc} 47 & & 33 & & 19 & & \underline{5} & & \underline{-9} & & \underline{-23} \\ & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \curvearrowright & & \\ & +(-14) & & +(-14) & & +(-14) & & +(-14) & & +(-14) & & \end{array}$$

common difference $d = -14$

Arithmetic Progression

Which of these are arithmetic progressions?

① 9, 16, 23, 29, 35, 40, 45, 49, 53, 56, 59, 61, 63, ... X
 $\underbrace{+7} \quad \underbrace{+7} \quad \underbrace{+6}$ no common difference

② 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, ... X
 $\underbrace{+2} \quad \underbrace{+4}$ no common difference

③ 111, 106, 101, 96, 91, 86, 81, 76, 71, 66, 61, ... ✓ common difference $d = -5$
 $\underbrace{+(-5)} \quad \underbrace{+(-5)} \quad \underbrace{+(-5)}$

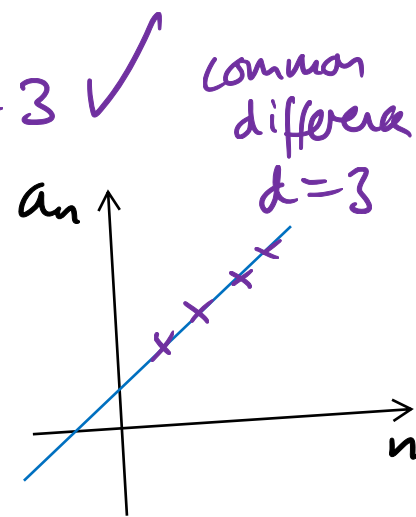
④ (a_n) given by $a_n = 3n + 2$ ↗ +3
 a_n, a_{n+1}

$$a_{n+1} - a_n = 3(n+1) + 2 - (3n + 2) = \cancel{3n} + 3 + \cancel{2} - \cancel{3n} - \cancel{2} = 3 \quad \checkmark$$

common difference $d = 3$

⑤ $x(0), x(1), x(2), x(3), \dots$ where $x(k) - x(k-1) = x(k+1) - x(k)$ for $k \geq 1$
 $\underbrace{+d} \quad \underbrace{+d} \quad \underbrace{+d}$

✓



Arithmetic Progression

What is the 100th term of the arithmetic progression 4, 7, 10, ... ?

$$\begin{array}{c} \curvearrowright \quad \curvearrowright \\ 4, 7, 10, \dots \\ +3 \quad +3 \end{array}$$

$$d = 3$$

$$\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ 4 & 7 & 10 & \checkmark \\ \curvearrowright & \curvearrowright & \curvearrowright & \\ +3 & +3 & +3 & \end{array}$$

$$a_2 = 4 + 3$$

$$a_3 = 4 + 3 + 3$$

$$a_4 = 4 + 3 + 3 + 3$$

$$a_n = 4 + \underbrace{3 + 3 + \dots + 3}_{n-1} = 4 + 3(n-1)$$

a_{100}

$$a_n = 4 + 3(n-1)$$

$\leftarrow a_1$ $\leftarrow d$

$$a_n = a_1 + d(n-1)$$

$$a_{100} = 4 + 3(100-1) = 4 + 3 \times 99 = 4 + 297 = \underline{301}$$

$$\begin{array}{l} n=3 \quad a_3 = 4 + 3(3-1) \\ \quad \quad = 4 + 3(2) \\ \quad \quad = 4 + 6 = 10 \checkmark \end{array}$$

Arithmetic Progression

Suppose that the 3rd and 15th terms of an arithmetic progression sum to -34 and the 6th and 20th terms sum to -58. What is the n^{th} term?

Let $(a_n)_{n=1}^{\infty}$ be the A.P. with common difference d . So, $a_n = a_1 + d(n-1)$.

$$\begin{cases} a_3 + a_{15} = -34 \\ a_6 + a_{20} = -58 \end{cases} \quad \begin{matrix} a_3 = a_1 + 2d, & a_{15} = a_1 + 14d, \\ a_6 = a_1 + 5d, & a_{20} = a_1 + 19d. \end{matrix}$$

$$\begin{cases} (a_1 + 2d) + (a_1 + 14d) = -34 \\ (a_1 + 5d) + (a_1 + 19d) = -58 \end{cases}$$

$$\begin{cases} 2a_1 + 16d = -34 & \text{--- ①} \\ 2a_1 + 24d = -58 & \text{--- ②} \end{cases}$$

$$\text{②} - \text{①}: \quad 8d = -24$$

$$d = -\frac{24}{8} = -3$$

$$\begin{array}{l} \text{Sub into ①:} \\ 2a_1 + 16(-3) = -34 \\ 2a_1 - 48 = -34 \\ 2a_1 = 14 \end{array}$$

$$a_1 = 7$$

$$\begin{aligned} \text{So } a_n &= a_1 + d(n-1) \\ &= 7 + (-3)(n-1) \\ &= 7 - 3n + 3 \end{aligned}$$

$$\underline{\underline{a_n = 10 - 3n}}$$

check

$$\begin{aligned} a_3 &= 10 - 3 \times 3 = 1 \\ a_{15} &= 10 - 3 \times 15 = -35 \\ a_3 + a_{15} &= \underline{\underline{-34}} \\ a_6 &= 10 - 3 \times 6 = -8 \\ a_{20} &= 10 - 3 \times 20 = -50 \end{aligned}$$