Geometric Progression

$3, 6, 12, 24, 48, 96, 192, 384, 768, 1536, 3072, 6144, 12\,288, 24\,576, 49\,152, \ldots$

$\frac{567}{320}, \frac{189}{80}, \frac{63}{20}, \frac{21}{5}, \frac{84}{15}, \frac{336}{45}, \ldots$

What are the next 4 terms of the geometric sequence that starts $20, 10, 5$?

Which of these are geometric progressions?

① $784, 392, 196, 98, 49, \ldots$

② $13, 26, 78, 312, 624, \ldots$

③ $1, -1, 1, -1, 1, -1, \ldots$

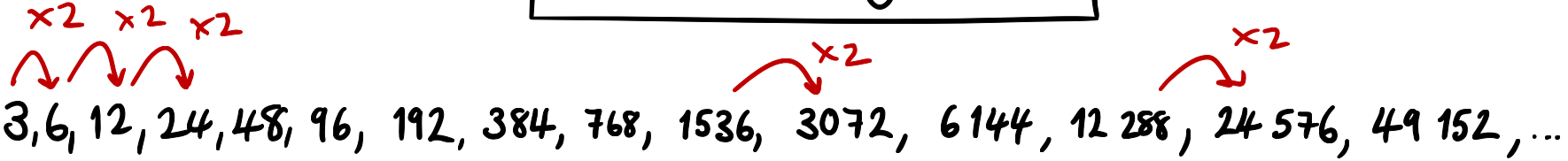
④ (a_n) given by $a_n = \frac{4 \times 5^{2n}}{3^n \times 7^{4n}}$.

⑤ $x(0), x(1), x(2), x(3), \ldots$ where $x(0) = 17$ & $x(k+1)x(k-1) = (x(k))^2$ for $k \geq 1$

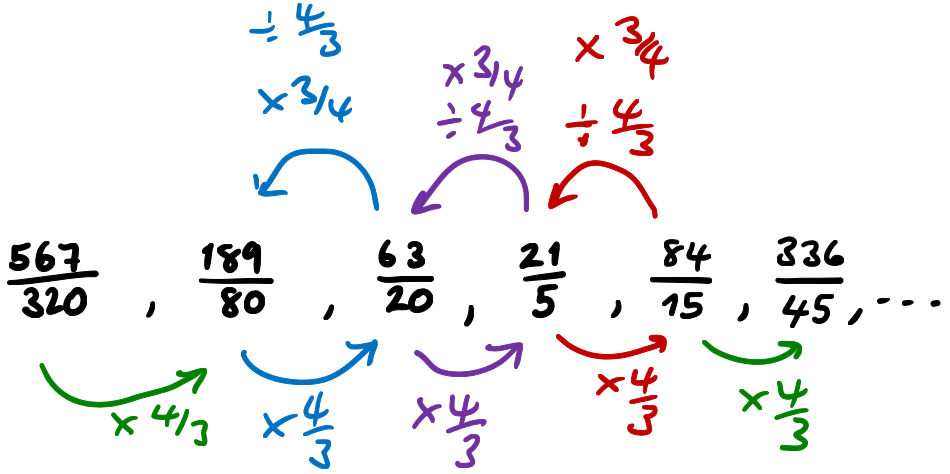
What is the 100^{th} term of the geometric progression $\frac{-1}{10}, \frac{-1}{2}, \frac{-5}{2}, \ldots$?

Suppose that $t_1, t_2, t_3, \ldots$ forms a geometric progression where $t_4 = 10$ and $t_7 = 80$. What is t_{11}?

Geometric Progression



common ratio $r = 2$



common ratio $\frac{4}{3}$

$\frac{189}{80} \times \frac{3}{4} = \frac{189 \times 3}{80 \times 4} = \frac{567}{320}$

$189a = 200a - 11$

$3 \times 189 = 600 - 33 = 567$

$\frac{21 \times 4}{5 \times 3} = \frac{84}{15}$

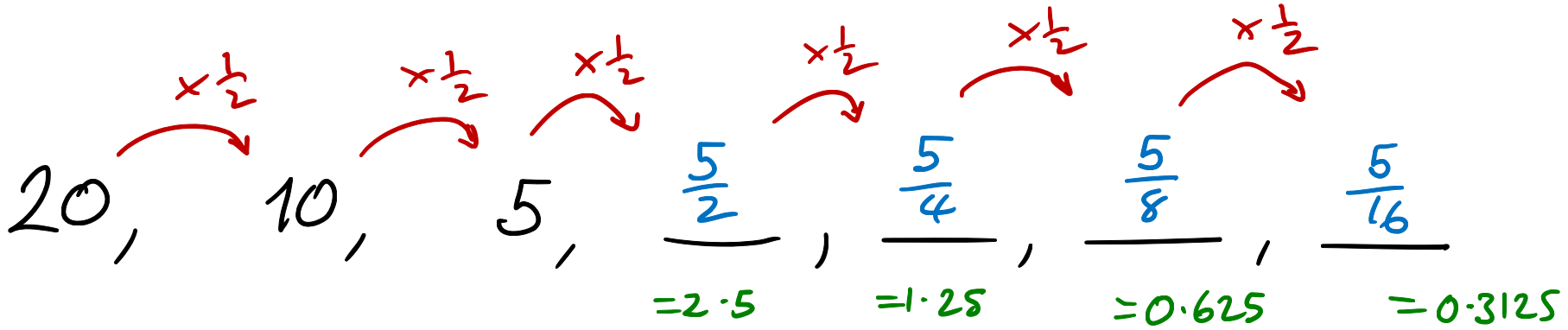
$\frac{84}{15} \times \frac{4}{3} = \frac{84 \times 4}{15 \times 3} = \frac{336}{45}$

$\frac{63}{20} \times \frac{3}{4} = \frac{63 \times 3}{20 \times 4} = \frac{189}{80}$

$\frac{21}{5} \times \frac{3}{4} = \frac{21 \times 3}{5 \times 4} = \frac{63}{20}$

Geometric Progression

What are the next 4 terms of the geometric sequence that starts 20, 10, 5 ?



$$r = \frac{a_2}{a_1} = \frac{10}{20} = \frac{1}{2}$$

$$r = \frac{a_3}{a_2} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{5}{2} \times \frac{1}{2} = \frac{5 \times 1}{2 \times 2} = \frac{5}{4}$$

Geometric Progression

Which of these are geometric progressions?

✓ ① 784, 392, 196, 98, 49, ...

×2 ×2 ×2 ×2

×½ ×½ ×½ ×½

$r = \frac{a_{n+1}}{a_n}$ $\quad r = \frac{a_5}{a_4} = \frac{49}{98} = \frac{1}{2}$

✗ ② 13, 26, 78, 312, 624, ...

×2 ×3 no common ratio

$\frac{a_2}{a_1} = \frac{26}{13} = \frac{2\times13}{13} = 2$, $\frac{a_3}{a_2} = \frac{78}{26} = 3$

✓ ③ 1, −1, 1, −1, 1, −1, ...

×(−1) ×(−1) ×(−1)

$r = \frac{a_2}{a_1} = \frac{-1}{1} = -1$ $\quad \frac{a_3}{a_2} = \frac{1}{-1} = -1$

✓ ④ (a_n) given by $a_n = \frac{4\times5^{2n}}{3^n \times 7^{4n}}$. $\frac{a_{n+1}}{a_n} = a_{n+1}\frac{1}{a_n} = \frac{4\times5^{2n+2}}{3^{n+1}7^{4n+4}}\frac{3^n 7^{4n}}{4\times5^{2n}} = \frac{5^2}{3\times7^4}$

✓ ⑤ $x(0), x(1), x(2), x(3), \ldots$ where $x(0) = 17$ & $x(k+1)x(k-1) = (x(k))^2$ for $k \geq 1$

×r ×r ×r

$x(k-1)$ $x(k)$ $x(k+1)$

$\frac{x(k)}{x(k-1)} = \frac{x(k+1)}{x(k)}$ $\quad \frac{x(k+1)}{x(k)} = \frac{x(k)}{x(k-1)}$

$\frac{3^n}{3^{n+1}} = 3^{n-(n+1)} = 3^{-1} = \frac{1}{3}$

Geometric Progression

What is the 100th term of the geometric progression $-\frac{1}{10}, -\frac{1}{2}, -\frac{5}{2}, \ldots$?

$a_1, a_2, a_3, \ldots$

common ratio $r = \frac{a_2}{a_1} = \frac{-1/2}{-1/10} = \left(-\frac{1}{2}\right)\left(\frac{10}{-1}\right) = \frac{10}{2} = 5$, $\frac{a_3}{a_2} = \frac{-5/2}{-1/2} = \frac{5\times(-1/2)}{1\times(-1/2)} = 5$

$a_1 = -\frac{1}{10}$

$a_2 = -\frac{1}{10} \times 5$

$a_3 = \left(-\frac{1}{10} \times 5\right) \times 5$

$a_4 = \left(-\frac{1}{10} \times 5 \times 5\right) \times 5$

$a_n = -\frac{1}{10} \times \underbrace{5\times5\times5\times\cdots\times5}_{n-1 \text{ times}}$

$= -\frac{1}{10} \times 5^{n-1}$

$a_{100} = -\frac{1}{10} \times 5^{199} = \frac{-5^{199}}{10} = \frac{-5^{198}}{2}$

a_1

$a_2 = a_1 r$

$a_3 = a_2 r = (a_1 r) r = a_1 r^2$

$a_4 = a_3 r = (a_1 r^2) r = a_1 r^3$

$\vdots$

$a_n = a_1 r^{n-1}$

Geometric Progression

Suppose that $t_1, t_2, t_3, \ldots$ forms a geometric progression where $t_4 = 10$ and $t_7 = 80$. What is t_{11}?

Let r be the common ratio. So, $\boxed{t_n = t_1 r^{n-1}}$

$t_4 = 10$

$t_1 r^3 = 10$

$t_7 = 80$

$t_1 r^6 = 80$

$8 = \frac{80}{10} = \frac{t_1 r^6}{t_1 r^3} = r^3$

$r = \sqrt[3]{8} = 2$

$t_1 = \frac{10}{r^3}$ $\quad t_1 = \frac{80}{r^6}$

$\frac{10}{r^3} = \frac{80}{r^6} \Rightarrow 10r^6 = 80r^3$

$\Rightarrow r^6 = 8r^3$

$\Rightarrow r^3 = 8$

$t_1 r^3 = 10$

$t_1 2^3 = 10$

$8t_1 = 10$

$t_1 = \frac{10}{8} = \frac{5}{4}$

$t_{11} = t_1 r^{10} = \frac{5}{4} 2^{10} = \frac{5}{2^2} 2^{10} = 5 \times 2^8$

$= 5 \times 2 \times 2^7$

$= 10 \times 128$

$= 1280$

Geometric Progression

(x_n) G.P. common ratio r

$$x_n = x_1 r^{n-1}$$