Series, Sums & Sigma Notation

Calculate $\sum_{j=2}^{7} j^2$

Find a simple formula (in terms of n) for $\sum_{k=1}^{n} \frac{1}{k(k+1)}$

What is the value of $\sum_{i=1}^{10} a_i$ where $a_i = 3+2i$ for each i

Which is the odd one out?

$$\sum_{i=1}^{7} (i+1)^3 \quad , \quad \sum_{k=2}^{8} k^3 \quad , \left(\sum_{n=2}^{7} n^3\right) + 8^3, \quad \sum_{j=3}^{9} (j^3 - 3j^2 + 3j - 1), \quad \left(\sum_{\ell=2}^{8} \ell\right)^3$$

Write out the sum $\frac{6}{9} + \frac{7}{16} + \frac{8}{25} + \cdots + \frac{20}{289}$ using Σ-notation

Find the value of $\sum_{n=1}^{\infty} \frac{9}{10^n}$

Series, Sums & Sigma Notation

summands

$$\underbrace{(3+5)+7+(9+11)}_{\text{sum}} = \underbrace{35}_{\text{sum}}$$

$$a_1 + a_2 + a_3 + a_4 + \dots + a_{20}$$

index

$$x(0) + x(1) + x(2) + \dots + x(n)$$

limit

$$\overbrace{2+3+0+0+0+0+\dots}^{\text{convergent}} = 5$$

$$y_1 + y_2 + y_3 + y_4 + \dots$$

series

$$\underbrace{1+2+3+4+5+\dots}_{\text{divergent}} = \infty$$

Sigma Σ

index range

$$\sum_{n=1}^{4} z_n = \sum_{n=1,2,\dots,4} z_n = \sum_{n=1}^{4} z_n = z_1 + z_2 + z_3 + z_4$$

index

$$\sum_{k=0}^{5} 3^{2k} = 3^{2\times 0} + 3^{2\times 1} + 3^{2\times 2} + 3^{2\times 3} + 3^{2\times 4} + 3^{2\times 5} = 3^0 + 3^2 + 3^4 + 3^6 + 3^8 + 3^{10}$$

double sum

$$\sum_{j=0}^{2}\left(\sum_{i=1}^{3} a_{ij}\right) = \sum_{i=1}^{3} a_{i0} + \sum_{i=1}^{3} a_{i1} + \sum_{i=1}^{3} a_{i3} = (a_{10} + a_{20} + a_{30}) + (a_{11} + a_{21} + a_{31}) + (a_{13} + a_{23} + a_{33})$$

Series, Sums & Sigma Notation

Calculate $\sum_{j=2}^{7} j^2$

$$= 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$$

$$= 4 + 9 + 16 + 25 + 36 + 49$$

$$= 139$$

Series, Sums & Sigma Notation

Find a simple formula (in terms of n) for $\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} + \dots + \frac{1}{n(n+1)}$

$$S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{n(n+1)}$$

$$S_1 = \frac{1}{2}$$

$$S_2 = \frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$S_3 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{2}{3} + \frac{1}{12} = \frac{8}{12} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$$

$$S_4 = \underline{\frac{1}{2} + \frac{1}{6} + \frac{1}{12}} + \frac{1}{20} = 3/4 + \frac{1}{20} = \frac{15}{20} + \frac{1}{20} = \frac{16}{20} = \frac{4}{5}$$

$$\boxed{S_n = \frac{n}{n+1}}$$

if $S_{n-1} = \dfrac{n-1}{n}$, then

$$S_n = S_{n-1} + \frac{1}{n(n+1)} = \frac{n-1}{n} + \frac{1}{n(n+1)}$$

$$= \frac{(n-1)(n+1)}{n(n+1)} + \frac{1}{n(n+1)}$$

$$= \frac{n^{\cancel{2}} \cancel{-1} \cancel{+1}}{\cancel{n}(n+1)} = \frac{n}{n+1}$$

Series, Sums & Sigma Notation

What is the value of $\sum_{i=1}^{10} a_i$ where $a_i = 3 + 2i$ for each i

$$\sum_{i=1}^{10} a_i = \sum_{i=1}^{10} (3+2i) = \begin{aligned} & 3+2\times 1 \\ + & 3+2\times 2 \\ + & 3+2\times 3 \\ & \vdots \\ + & 3+2\times 10 \end{aligned} = 3\times 10 + 2(\underbrace{1+2+3+\dots+10}_{=55})$$

$$\sum_{i=1}^{10} a_i = a_1 + a_2 + a_3 + \dots + a_{10} = 5+7+9+\dots+23 = 140$$

$$\sum_{i=1}^{10}(3+2i) = 3\underbrace{\sum_{i=1}^{10} 1}_{=10} + 2\underbrace{\sum_{i=1}^{10} i}_{=55} = 3\times 10 + 2\times 55$$

$$= 30 + 110$$

$$= \underline{\underline{140}}$$

Series, Sums & Sigma Notation

Which is the odd one out?

$$\sum_{i=1}^{7} (i+1)^3 = 2^3 + 3^3 + \cdots + 8^3$$

$$\sum_{k=2}^{8} k^3 = 2^3 + 3^3 + \cdots + 8^3$$

$$\left(\sum_{n=2}^{7} n^3\right) + 8^3 = \left(2^3 + 3^3 + \cdots + 7^3\right) + 8^3$$

$$\sum_{j=3}^{9} \underbrace{(j^3 - 3j^2 + 3j - 1)}_{=(j-1)^3} = \sum_{j=3}^{9} (j-1)^3 = 2^3 + 3^3 + \cdots + 8^3$$

$$\left(\sum_{l=2}^{8} l\right)^3 = \left(2 + 3 + \cdots + 8\right)^3 \longleftarrow \text{'odd one out'}$$

Series, Sums & Sigma Notation

Write out the sum $\frac{6}{9} + \frac{7}{16} + \frac{8}{25} + \cdots + \frac{20}{289}$ using Σ-notation

$+1 \quad +1 \quad +1$

$3^2 \quad 4^2 \quad 5^2 \quad 17^2$

$n = 6 \quad 7 \quad 8 \quad 20$

$j = 3 \quad 4 \quad 5 \quad 17$

$k = 1 \quad 2 \quad 3 \quad 15$

$$\sum_{n=6}^{20} \frac{n}{(n-3)^2}$$

$$\sum_{j=3}^{17} \frac{j+3}{j^2}$$

$$\sum_{k=1}^{15} \frac{k+5}{(k+2)^2}$$

Series, Sums & Sigma Notation

Find the value of $\sum_{n=1}^{\infty} \frac{9}{10^n} = \frac{9}{10^1} + \frac{9}{10^2} + \frac{9}{10^3} + \cdots$

$$= \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots$$

$$= \lim_{k\to\infty} \Big(\underbrace{\frac{9}{10} + \frac{9}{100} + \cdots + \frac{9}{10^k}}_{= S_k = \sum_{n=1}^{k} \frac{9}{10^n}} \Big) = 1$$

$$0.9 = S_1 = \frac{9}{10} = \frac{10^1 - 1}{10^1}$$

$$0.99 = S_2 = \frac{9}{10} + \frac{9}{100} = \frac{90}{100} + \frac{9}{100} = \frac{99}{100} = \frac{10^2 - 1}{10^2}$$

$$0.999 = S_3 = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} = \frac{99}{100} + \frac{9}{1000} = \frac{990}{1000} + \frac{9}{1000} = \frac{999}{1000} = \frac{10^3 - 1}{10^3}$$

$$S_k = \frac{10^k - 1}{10^k} = \frac{1 - \frac{1}{10^k}}{1} = 1 - \underbrace{\left(\frac{1}{10}\right)^k}_{\to 0} \longrightarrow 1$$

$$\underline{0.999\ldots = 1}$$