Arithmetic Series

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Calculate $5 + 8 + 11 + 14 + 17 + 20 + 23$

Simplify $1 + 2 + 3 + 4 + 5 + 6 + \cdots + n$

$$\sum_{k=0}^{35} \left(\frac{k}{2} + 1\right)$$

Find the sum of the first 50 terms of $32 + 27 + 22 + 17 + 12 + \cdots$

Compute the value of $3 + 5 + 7 + \cdots + 401$

How many terms of the arithmetic progression $11 + 20 + 29 + \cdots$ do we need to add to get a sum over 180 000 ?

Arithmetic Series

$$\overset{+3}{\curvearrowright}\ \overset{+3}{\curvearrowright}\ \overset{+3}{\curvearrowright}\ \overset{+3}{\curvearrowright}\ \overset{+3}{\curvearrowright}\ \overset{+3}{\curvearrowright}$$

Calculate $\underset{a_1}{\underline{\underline{5}}} + 8 + 11 + 14 + 17 + 20 + \underline{\underline{23}}$

common difference $d = 3$

initial term $a_1 = 5$

terms $n = 7$

final term $a_n = a_7 = 23$

$$x = 5 + 8 + 11 + 14 + 17 + 20 + 23$$

$$x = 23 + 20 + 17 + 14 + 11 + 8 + 5$$

$$2x = 28 \times 7$$

$$x = \frac{28 \times 7}{2} = 14 \times 7 = \underline{\underline{98}}$$

$$x = \frac{(5 + 23) \times 7}{2} = \frac{(a_1 + a_n)}{2} \times n$$

Arithmetic Series

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Simplify $\quad 1 \overset{+1}{+} 2 \overset{+1}{+} 3 \overset{+1}{+} 4 + 5 + 6 + \cdots + n \quad = n\left(\frac{1+n}{2}\right)$

<u>check</u> $\quad$ $n=3$: $1+2+3=6$

$$3\left(\frac{1+3}{2}\right) = 3\left(\frac{4}{2}\right) = 3\times 2$$
$$= 6 \checkmark$$

$n=5$: $1+2+3+4+5 = 15$

$$5\left(\frac{6}{2}\right) = 5\times 3 = 15 \checkmark$$

Arithmetic Series

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

$$\sum_{k=0}^{35}\left(\frac{k}{2}+1\right) = 1 \overset{+0.5}{+} 1.5 \overset{+0.5}{+} 2 \overset{+0.5}{+} 2.5 + \cdots + 17.5 \qquad n=36$$

$$= 36 \times \frac{1+17.5}{2}$$

$$= 36 \times \frac{18.5}{2}$$

$$= 36 \times \frac{37/2}{2}$$

$$= \frac{36 \times 37}{4}$$

$$= 9 \times 37 = (10-1) \times 37 = 10 \times 37 - 37$$

$$= 370 - 37 = \underline{\underline{333}}$$

Arithmetic Series

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Find the sum of the first 50 terms of $32 + 27 + 22 + 17 + 12 + \cdots$

$-5 \quad -5 \quad -5 \quad -5$

$d = -5$

$a_1 = 32$

$n = 50$

$a_n = a_{50}$

$$a_k = a_1 + (k-1)d$$

$$\begin{aligned} a_n = a_{50} &= a_1 + (50-1)d \\ &= 32 + 49 \times (-5) \\ &= 32 - 245 \\ &= -213 \end{aligned}$$

$$\begin{aligned} 32 + 27 + \cdots + (-213) &= 50 \times \left(\frac{32 + (-213)}{2}\right) \\ &= 50 \times \frac{-181}{2} = \underline{\underline{-4\,525}} \end{aligned}$$

Arithmetic Series

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Compute the value of $3 \overset{+2}{+} 5 \overset{+2}{+} 7 \overset{+2}{+} \cdots + 401$

$d = 2$

$a_1 = 3$

$a_n = 401$

$n = 200$

$$a_k = a_1 + (k-1)d$$

$$a_n = a_1 + (n-1)d$$

$$401 = 3 + (n-1) \times 2$$

$$398 = (n-1) \times 2$$

$$\frac{398}{2} = n-1$$

$$199 = n-1$$

$$\underline{200 = n}$$

$$\text{Answer} = n\left(\frac{a_1 + a_n}{2}\right)$$

$$= 200\left(\frac{3 + 401}{2}\right)$$

$$= 200 \times \frac{404}{2}$$

$$= 100 \times 404$$

$$= 40{,}400$$

Arithmetic Series

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

How many terms of the arithmetic progression $\underline{11} + \overset{+9}{20} + \overset{+9}{29} + \cdots$ do we need to add to get a sum over 180 000 ?

$d = 9$

$a_1 = 11$

Let n be the smallest value with $S_n \geqslant 180\,000$

$$a_n = a_1 + (n-1)d$$
$$a_n = 11 + (n-1)9$$
$$= 11 + 9n - 9$$
$$= 9n + 2$$

$$S_n = n\left(\frac{a_1 + a_n}{2}\right) = n\left(\frac{11 + a_n}{2}\right)$$

$$S_n = n\left(\frac{9n + 13}{2}\right)$$

$$S_n \geqslant 180\,000$$
$$\Leftrightarrow \quad n\left(\frac{9n+13}{2}\right) \geqslant 180\,000$$
$$\Leftrightarrow \quad n(9n+13) \geqslant 360\,000$$
$$\Leftrightarrow \quad 9n^2 + 13n \geqslant 360\,000$$

$$n^2 + \frac{13}{9}n \geqslant 40\,000$$
$$\left(n + \frac{13}{18}\right)^2 - \left(\frac{13}{18}\right)^2 \geqslant 40\,000$$

$$n^2 \geqslant 40\,000 = 2^2 \times 100^2 = (200)^2$$

$$n \approx 200$$

$$S_{200} = 200 \times \left(\frac{1800 + 13}{2}\right) = 100 \times 1813 = 181{,}300 \checkmark$$

$$S_{199} = 199 \times \left(\frac{9 \times 199 + 13}{2}\right) = 179{,}498 \times$$