Geometic Series

$$\sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}$$

Simplify $1 + x + x^2 + x^3 + \cdots + x^{n-1}$

Calculate $1 + 2 + 4 + 8 + 16 + 32$

Find the sum of the first 20 terms of $5 + 15 + 45 + 135 + 405 + \cdots$

Find $\sum_{k=0}^{10} a_k$ where $a_0 = 5$ and $a_n = \frac{2}{3} a_{n-1}$ for $n \geq 1$

Find the value of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

Geometic Series

Simplify $1 + x + x^2 + x^3 + \cdots + x^{n-1}$

$$A = 1 + x + x^2 + \cdots + x^{n-2} + x^{n-1}$$

$$-Ax = \quad -x - x^2 - x^3 - \cdots \quad - x^{n-1} - x^n$$

$$A - Ax = 1 - x^n$$

$$A(1-x) = 1 - x^n$$

$$A = \frac{1-x^n}{1-x} \quad \text{if } x \neq 1$$

if $x = 1$,

$$A = 1 + x + x^2 + \cdots + x^{n-1}$$

$$= 1 + 1 + 1 + \cdots + 1 = n$$

Geometic series a initial term, r = common ratio, n terms

$$ar^0 + ar^1 + ar^2 + ar^3 + \cdots + ar^{n-1} = a(1 + r + r^2 + \cdots + r^{n-1})$$

$$\sum_{k=0}^{n-1} ar^k = \begin{cases} a\,\frac{1-r^n}{1-r} & \text{if } r \neq 1 \\ an & \text{if } r = 1 \end{cases}$$

Geometic Series

$$\sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}$$

Calculate $1+2+4+8+16+32$ (×2 ×2 ×2 ×2 ×2)

initial term $a=1$

common ratio $r=2$

terms $n=6$

$$= \frac{a(1-r^n)}{1-r} = \frac{1\times(1-2^6)}{1-2} = \frac{1-2^6}{-1} = \frac{1-64}{-1} = \frac{-63}{-1}$$

$$= 63$$

$$1+2+4+8+16+32$$

$$= 3 + 20 + 40 = 63 \checkmark$$

Geometic Series

$$\sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}$$

Find the sum of the first 20 terms of $5 + 15 + 45 + 135 + 405 + \cdots$

×3 ×3 ×3 ×3

$r = 3$
$a = 5$
$n = 20$

$$= \frac{5(1-3^{20})}{1-3}$$

$$= \frac{5(3^{20}-1)}{3-1} = \frac{5(3^{20}-1)}{2}$$

$$= 8,716,961,000$$

Geometic Series

$$\sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}$$

Find $\sum_{k=0}^{10} a_k$ where $a_0 = 5$ and $a_n = \frac{2}{3} a_{n-1}$ for $n \geq 1$

$n = 11$

$a_0 = 5$: initial term, $a = 5$

Common ratio $r = \frac{2}{3}$

$a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10}$

$$\sum_{k=0}^{10} a_k = \frac{5\left(1-\left(\frac{2}{3}\right)^{11}\right)}{1-\frac{2}{3}}$$

$$= \frac{5\left(1-\left(\frac{2}{3}\right)^{11}\right)}{\left(\frac{1}{3}\right)}$$

$$= 15\left(1-\left(\frac{2}{3}\right)^{11}\right)$$

$$\simeq 14.8265$$

Geometic Series

$$\sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}$$

Find the value of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 1$ (each step $\times \frac{1}{2}$)

common ratio $r = \frac{1}{2}$

$a = \frac{1}{2}$

Let S_n be the sum of the 1st n terms.

$$S_n = \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots + \left(\frac{1}{2}\right)^n$$

$$= \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

$$= \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(\frac{1}{2}\right)}$$

$$= 1-\left(\frac{1}{2}\right)^n$$

$\rightarrow 1 - 0 = 1$