Matrices: Notation & Terminology

$\begin{pmatrix} 1 & 5 & 3 \\ 2 & 0 & -\frac{1}{2} \end{pmatrix}$, $\begin{bmatrix} 2 & 4 \\ 3 & -1 \\ 0 & 10 \end{bmatrix}$, $\begin{vmatrix} 2 & 5 \\ 3 & -6 \end{vmatrix}$

$A = (a_{ij})$, A_{ij}

$n \times m$

column vector

row vector

square

diagonal

upper triangular

lower triangular

A', A^T, $|A|$, $\det(A)$, $\mathrm{tr}(A)$

I_n identity matrix

$O_{n\times m}$ zero matrix

invertible, nonsingular, A^{-1}

Matrices: Notation & Terminology

$(1,2)^{th}$ entry

$$\begin{array}{r} \text{row } 1 \rightarrow \\ 2 \rightarrow \end{array} \begin{pmatrix} 1 & 5 & 3 \\ 2 & 0 & -\frac{1}{2} \end{pmatrix}, \qquad \begin{bmatrix} 2 & 4 \\ 3 & -1 \\ 0 & \boxed{10} \end{bmatrix} = A \qquad \cancel{\left\{ \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix} \right\}} \qquad \begin{vmatrix} 2 & 5 \\ 3 & -6 \end{vmatrix} = \text{determinant of } \begin{pmatrix} 2 & 5 \\ 3 & -6 \end{pmatrix}$$

column 1 2 3

A_{ij} $\qquad A_{3,2} = 10, \quad A_{2,1} = 3$

$$B = (\beta_{ij}) = \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1m} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2m} \\ \vdots & & \ddots & \vdots \\ \beta_{n1} & \beta_{n2} & \cdots & \beta_{nm} \end{pmatrix}$$

$n \times m$

A is a 3×2 matrix

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column vector $n \times 1$ matrix eg. $\begin{pmatrix} 5 \\ 2 \\ 6 \end{pmatrix} = a$ $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \underline{x}$

row vector $1 \times m$ matrix $\begin{pmatrix} 7 & 2 & 4 \end{pmatrix} = \underline{u}$ $(z_1, z_2, \ldots, z_m) = z$

square $n \times n$ matrix $\begin{pmatrix} 5 & 3 \\ 2 & 7 \end{pmatrix}$, $\begin{pmatrix} 10 & 7 & 2 \\ 0 & 9 & 5 \\ 1 & -2 & 6 \end{pmatrix}$

diagonal $a_{ij} = 0$ if $i \neq j$ $\begin{pmatrix} a_{11} & & 0 \\ & a_{22} & \\ 0 & & \ddots \; a_{nn} \end{pmatrix}$ eg $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, $\begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 7 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

upper triangular $a_{ij} = 0$ if $i > j$ $\begin{pmatrix} a_{11} & a_{12} & & \\ 0 & a_{22} & \cdots & \\ 0 & 0 & \ddots & \\ \vdots & \vdots & & a_{nm} \\ 0 & 0 & & \end{pmatrix}$ eg $\begin{pmatrix} 3 & 2 & 0 \\ 0 & 4 & 7 \\ 0 & 0 & 1 \end{pmatrix}$

lower triangular

$a_{ij} = 0$ if $i < j$ $\begin{pmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & 0 & \cdots & 0 \\ \vdots & & & \ddots & a_{nm} \end{pmatrix}$

Matrices: Notation & Terminology

I_n identity matrix

$n \times n$ matrix whose $(i,j)^{th}$ entry is $\begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$O_{n \times m}$ zero matrix

$$O_{2\times 3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$O_{2\times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$O = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Matrices: Notation & Terminology

A', A^T

If $A = (a_{ij})$ is an $n \times m$ matrix

Then the <u>transpose</u> of A, $A' = A^T = (a_{ji})$ is $m \times n$

$$A = \begin{pmatrix} 2 & 7 & 5 \\ 3 & 6 & 9 \end{pmatrix}, \quad A' = \begin{pmatrix} 2 & 3 \\ 7 & 6 \\ 5 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 7 \\ 10 & 0 \end{pmatrix}^T = \begin{pmatrix} 3 & 10 \\ 7 & 0 \end{pmatrix}$$

tr(A)

If $A = (a_{ij})$ is $n \times n$ matrix, then

the <u>trace</u> of A, $tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$

$$tr\begin{pmatrix} 3 & 7 & 9 \\ 2 & 4 & 6 \\ 0 & 1 & 5 \end{pmatrix} = 3+4+5 = 12$$

Matrices: Notation & Terminology

$|A|$, $\det(A)$

If A is a square matrix, then it has a <u>determinant</u>

$\det(A)$, $|A|$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

eg $X = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$,

$|X| = 3 \times 4 - 1 \times 2$
$= 12 - 2$
$= 10$

$\det(X) = 10$

$\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = 10$

invertible, nonsingular

If A is an $n \times n$ matrix.
Then A is <u>invertible</u> if there is some $n \times n$ matrix B with

$$AB = BA = I_n$$

A^{-1}

inverse $B = A^{-1}$