Matrix Multiplication

$\underline{x} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$, $\underline{y} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$, $A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 & 3 \\ 1 & 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 0 & 4 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A\underline{x}$

$(2A + 3B)\underline{x}$

AC

CA

CDA

Does ED equal DE?

$\underline{x}\,\underline{y}^T$

D^2

D^3

D^0

D^{-1}

D^{-2}

Matrix Multiplication

$$\underline{x} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}, \quad \underline{y} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

Inner product (a.k.a. scalar product / dot product)

$$\underline{x} \cdot \underline{y} = \langle \underline{x}, \underline{y} \rangle = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = 2 \times 0 + 5 \times 2 + (-1) \times 3$$

$$= 0 + 10 - 3 = \underline{7}$$

$$\begin{pmatrix} 2 & 5 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \underline{x}^T \underline{y}$$

Matrix Multiplication

$$A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 0 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2\times2+5\times3+-1\times0 & 2\times5+5\times1+(-1)\times4 \\ 3\times2+0\times3+4\times0 & 3\times5+0\times1+4\times4 \end{bmatrix}$$

$(2\ 5\ -1)\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ $(2\ 5\ -1)\begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix}$ $(3\ 0\ 4)\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ $(3\ 0\ 4)\begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix}$

2 × 2

$$= \begin{bmatrix} 19 & 11 \\ 6 & 31 \end{bmatrix}$$

$$CA = \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2\times2+5\times3 & 2\times5+5\times0 & 2\times(-1)+5\times4 \\ 2\times3+3\times1 & 3\times5+1\times0 & 3\times(-1)+1\times4 \\ 0\times2+4\times3 & 0\times5+4\times0 & 0\times(-1)+4\times4 \end{bmatrix}$$

3 × 3

$$= \begin{bmatrix} 19 & 10 & 18 \\ 9 & 15 & 1 \\ 12 & 0 & 16 \end{bmatrix}$$

Matrix Multiplication

$$A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 2 & 3 \\ 1 & 4 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 5 & 2 & 3 \\ 1 & 4 & 0 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

AB is not defined!

$$XY = \begin{pmatrix} \quad \end{pmatrix}_{n \times m} \begin{pmatrix} \quad \end{pmatrix}_{m \times k} = \begin{pmatrix} \quad \end{pmatrix}_{n \times k}$$

X is $n \times m$

Y is $m \times k$

$\Rightarrow$ XY is $n \times k$

Matrix Multiplication

$$D = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Does ED equal DE?

$$ED = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 3 \end{bmatrix}$$

$1\times2+1\times1$

$$DE = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$$

$2\times1+4\times0$

No, $ED \neq DE$.

Matrix Multiplication

$$A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 0 & 4 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

CDA

$$(CD)A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 23 \\ 7 & 15 \\ 4 & 12 \end{bmatrix} \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 87 & 45 & 83 \\ 59 & 35 & 53 \\ 44 & 20 & 44 \end{bmatrix}$$

$$C(DA) = \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 16 & 10 & 14 \\ 11 & 5 & 11 \end{bmatrix} = \begin{bmatrix} 87 & 45 & 83 \\ 59 & 35 & 53 \\ 44 & 20 & 44 \end{bmatrix}$$

Matrix Multiplication

$$A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 2 & 3 \\ 1 & 4 & 0 \end{bmatrix}, \quad \underline{x} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix},$$

$$A\underline{x} = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 0 & 4 \end{bmatrix} \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 30 \\ 2 \end{pmatrix} \quad (2\ 5\ -1)\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} = 4 + 25 + 1 = 30$$

$$(2A + 3B)\underline{x} = \begin{pmatrix} 19 & 16 & 7 \\ 9 & 12 & 8 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 111 \\ 70 \end{pmatrix}$$

$$2(A\underline{x}) + 3(B\underline{x}) = 2\begin{pmatrix} 30 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 17 \\ 22 \end{pmatrix} = \begin{pmatrix} 111 \\ 70 \end{pmatrix}$$

$$(2A + 3B)\underline{x} = 2(A\underline{x}) + 3(B\underline{x})$$

$DE \neq ED$

$AC \neq CA$

Matrix Multiplication

$$D^4 = DDDD = D^3D = DD^3 = D^2D^2 = (D^2)^2$$

$$D^3 = DDD = D^2D = \begin{bmatrix} 8 & 20 \\ 5 & 13 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 36 & 92 \\ 23 & 59 \end{bmatrix}$$

$$D^2 = DD = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 20 \\ 5 & 13 \end{bmatrix}$$

$$D^1 = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = D$$

$$D^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$DI_n = D$

$$D^{-1} = \begin{pmatrix} 3/2 & -2 \\ -1/2 & 1 \end{pmatrix}$$

inverse of D

$$\begin{pmatrix} 3/2 & -2 \\ -1/2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 = D^0$$

$$D^{-2} = (D^2)^{-1} = (D^{-1})^2$$

$3^4 = 81$ ↑ 3

$3^3 = 27$ ↑ ×3

$3^2 = 9$ ↑ ×3

$3^1 = 3$ ↑ ×3

$3^0 = 1$

$3^{-1} = \frac{1}{3}$ ↑ ×3

$3^{-2} = \frac{1}{3^2} = (\frac{1}{3})^2$