Matrices & Linear Equations

$$\begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 7 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 2 & 3 & 4 \\ 1 & -2 & 9 \\ 3 & 7 & 1 \end{array}\right)$$

$$\begin{cases} 3x + 2y - 7z = 6 \\ 2x - 5y + z = -3 \\ x - y + 4z = 5 \end{cases}$$

$$\begin{cases} 3x_1 + 2x_2 - 4x_3 = -8 \\ 4x_1 - 2x_3 = 2 - 3x_2 \end{cases}$$

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$$

$$\begin{cases} 3x + 2y = 5 \\ 5x + 7y = 1 \end{cases}$$

Matrices & Linear Equations

$$\begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2a+3b \\ a-2b \\ 3a+7b \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix} \Leftrightarrow \begin{cases} 2a+3b=4 \\ 1a-2b=9 \\ 3a+7b=1 \end{cases}$$

$$A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 7 \end{pmatrix}$$ coefficient matrix

$$\underline{x} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\underline{y} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$$

$$A\underline{x} = \underline{y}$$

$$\left(\begin{array}{cc|c} 2 & 3 & 4 \\ 1 & -2 & 9 \\ 3 & 7 & 1 \end{array}\right)$$ augmented matrix

Matrices & Linear Equations

$$\begin{cases} 3x + 2y - 7z = 6 \\ 2x - 5y + z = -3 \\ x - y + 4z = 5 \end{cases}$$

$$\underbrace{\begin{pmatrix} 3 & 2 & -7 \\ 2 & -5 & 1 \\ 1 & -1 & 4 \end{pmatrix}}_{\text{coefficient matrix}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix}$$

augmented matrix $\left(\begin{array}{ccc|c} 3 & 2 & -7 & 6 \\ 2 & -5 & 1 & -3 \\ 1 & -1 & 4 & 5 \end{array}\right)$

Matrices & Linear Equations

$$\begin{cases} 3x_1 + 2x_2 - 4x_3 = -8 \\ 4x_1 - 2x_3 = 2 - 3x_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} 3x_1 + 2x_2 - 4x_3 = -8 \\ 4x_1 + 3x_2 - 2x_3 = 2 \end{cases}$$

$$\Leftrightarrow \begin{pmatrix} 3 & 2 & -4 \\ 4 & 3 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -8 \\ 2 \end{pmatrix}$$

Matrices & Linear Equations

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} \iff \begin{cases} 2p + 3q = 1 \\ p + 2q = 0 \\ 3p - 4q = 5 \end{cases}$$

Matrices & Linear Equations

$$\begin{cases} 3x + 2y = 5 \\ 5x + 7y = 1 \end{cases} \iff \underbrace{\begin{pmatrix} 3 & 2 \\ 5 & 7 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\underline{u}} = \underbrace{\begin{pmatrix} 5 \\ 1 \end{pmatrix}}_{\underline{w}}$$

$x = 3,\ y = -2$

$$A\underline{u} = \underline{w}$$

$$A^{-1}A = I_2$$

$$\underline{u} = I_2\underline{u} = A^{-1}A\underline{u} = A^{-1}\underline{w}$$

check

$3x + 2y = 9 - 4 = 5$ ✓

$5x + 7y = 15 - 14 = 1$ ✓

$$\det(A) = 3 \times 7 - 2 \times 5 = 21 - 10 = 11$$

$$A^{-1} = \frac{1}{11}\begin{pmatrix} 7 & -2 \\ -5 & 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\underline{w} = \frac{1}{11}\begin{pmatrix} 7 & -2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 35 - 2 \\ -25 + 3 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 33 \\ -22 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$