INTRODUCTION TO
NATURAL RESOURCE ECONOMICS

Lecture 2

Non-renewable resource exploitation: basic models

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1 Introduction

In Lecture 1, we saw that there was a general rule for the efficient exploitation of a non-renewable resource over time, which was (in discrete time notation)

$$v_t'(q_t) = \frac{1}{1+\delta}\lambda_{t+1}, \tag{1}$$

where

$$\lambda_{t+1} = \frac{1}{1+\delta}\lambda_{t+2}. \tag{2}$$

The rule states that we should extract now as long as the current marginal value of extraction is greater than the (discounted) shadow price of the resource in the next period. This shadow price is related to the shadow price in the subsequent period (and so on). Since we can equally well write (1) as

$$v_{t+1}'(q_{t+1}) = \frac{1}{1+\delta}\lambda_{t+2},$$

we can see that

$$v_t'(q_t) = \frac{1}{1+\delta}v_{t+1}'(q_{t+1}),$$

that is, the shadow price in the next period is just the marginal value of extraction in the next period. In choosing whether to extract now or in the next period, we look at whether the rate of increase in the (marginal) value of the resource is less than or greater than the interest (discount) rate. If it is less, we should extract now. If it is greater, it is optimal to defer extraction. This is a straightforward *intertemporal efficiency* question.

If there is an optimal and efficient extraction *path* over time, then the value of the resource must be rising at the interest rate. This is Hotelling's Rule, which, in its simplest form, we expressed in continuous time as

$$\frac{\dot{p}}{p(t)} = r,$$

where $p(t)$ is the price (value) of the resource. We can also express (1) and (2) in continuous time terms as

$$p(t) = \lambda(t) \tag{3}$$

and

$$\frac{\dot{\lambda}}{\lambda(t)} = r. \tag{4}$$

If we set up the optimisation problem in a particular way (by specifying a type of Lagrangian function called a *Hamiltonian*) we can obtain (3) and (4) directly from the FOCs for the *control* variable (here, the quantity extracted) and the so-called *state* variable (the size of the resource stock).

2 Hotelling's Rule in competitive markets

In a world of constant prices (and zero extraction costs), Hotelling's Rule as a description of extraction over time doesn't really make much sense. If prices aren't rising then it will always be optimal to extract now rather than later, and if extraction is costless it will be optimal to extract *all* of the resource now (and invest the proceeds elsewhere).

One obvious way in which we can have rising prices is with a downward-sloping demand curve and decreasing supply. An individual *competitive* firm will take instantaneous prices as given, but since all firms have an incentive to bring forward extraction if prices are rising more slowly than the interest rate and to delay extraction if prices are rising more rapidly than the interest rate, at market equilibrium Hotelling's Rule must hold. Why? Assuming that all firms have similar expectations about future prices, increasing extraction now implies lower current prices and hence a faster (expected) rate of price increase. Conversely, delaying extraction implies higher current prices and hence the expectation of a slower rate of price increase. This self-correction means that the rate of price increase converges to the interest rate r.

A competitive market is therefore characterised by the price path

$$\dot{p} = rp(t) > 0.$$

Then, as we saw previously, we can write any current price as a function of the initial price as

$$p(t) = p(0)\, e^{rt},$$

where $p(0)$ is the price at $t = 0$. We can also write any current price as a function of a final price $p(T)$, if we know it, as

$$p(t) = p(T)\, e^{-r[T-t]},$$

which is just $p(T)$ discounted back to the current time t. We can give some economic meaning to the end price $p(T)$ in the case of many non-renewable resources (fuels, in particular) by supposing that there exists a substitute for the resource - a *backstop* resource or technology - which is available at a higher price. At some price (the "backstop" or "choke" price) demand for the old resource falls to zero and the market is supplied entirely by the backstop resource.

If Hotelling's Rule holds, then, we have an aggregate extraction path characterised by rising prices and (hence) decreasing quantities extracted, ending at the backstop price at which demand and therefore extraction is zero.

A *social planner*, seeking to maximise total social welfare from resource use, would maximise the discounted sum of consumers' and producers' surplus over time, subject to the resource constraint. With zero extraction costs, this is equivalent to maximising the total area under the inverse demand curve

$$W(q(t)) \equiv \int_0^{q(t)} p(q(t))\, dq,$$

at any point in time, where $p(q(t))$ is the inverse demand curve for the resource. The competitive firm's objective is to maximise its own profit function (subject to the same resource constraint), which, with no extraction costs, we can write as

$$\pi(t) \equiv p(t)\, q(t).$$

Since

$$\frac{dW(q(t))}{dq} = \frac{\partial \pi(t)}{\partial q} = p(\cdot),$$

we can see straightaway that profit maximisation by competitive firms will also result in the maximisation of social welfare, *provided* that the interest rate faced by the industry reflects the social discount rate. Thus, given certain assumptions, the competitive outcome is also the socially optimal outcome.

3 Extraction by a monopoly

A monopoly producer seeks to maximise the same objective function as a competitive firm, that is

$$\int_0^T p\left(q\left(t\right)\right)q\left(t\right)e^{-rt}dt,$$

but its output affects the market price and it therefore has a downward-sloping marginal revenue curve, which we can write as

$$R_q \equiv \frac{d}{dq}\left[p\left(q\left(t\right)\right)q\left(t\right)\right] = p\left(\centerdot\right) + \frac{dp\left(\centerdot\right)}{dq}q\left(t\right) < p\left(\centerdot\right).$$

It is not difficult to see that the rule for an efficient extraction path by a monopoly will be

$$\frac{\dot{R}_q}{R_q} = r. \tag{5}$$

In the case of a monopoly, we do not need to assume that the individual firm has an expectation about the (exogenous) rate of change of the resource price. The monopolist *controls* the price by its choice of the quantity to extract at any point in time. The implication of (5) is that the firm schedules extraction so that its marginal revenue rises at the interest rate; equivalently, so that its discounted marginal revenue is constant through time.

Note that marginal revenue (and hence the shadow price of the resource stock) is always less than the market price, *except* at the end of the extraction period when $q\left(T\right) = 0$ and hence $R_q = p\left(T\right)$, the backstop price. It then follows that if the discounted marginal revenue is constant through time, the discounted market price must be *decreasing* over time.

For a given resource stock and market demand, both monopoly and competitive extraction end up at the same backstop price, so how do the extraction paths differ between a monopolist and a competitive firm?

Firstly, if the discounted monopoly price is decreasing over time, the current price must be increasing at a rate less than the interest rate, and therefore less than the rate at which it would increase under competitive extraction, i.e.,

$$\frac{\dot{R}_q}{R_q} = r \quad \Rightarrow \quad \frac{\dot{p}}{p\left(t\right)} < r.$$

Secondly, if the discounted monopoly price is decreasing over time, the initial monopoly price must be higher than the initial competitive price.

Thirdly, we know that, for a given market demand curve, higher prices mean lower quantities and *vice-versa*, so we can conclude that the monopolist begins by extracting smaller quantities than the competitive firm. We can also conclude that the rate at which extraction declines is slower under monopoly, so that the monopolist's extraction plan must extend over a longer period than that of the competitive firm. By implication, the monopolist is then extracting greater quantities than the competitive firm towards the end of its planning period.

So, monopoly extraction is more gradual and extended compared with competitive extraction, but this doesn't mean that monopoly extraction is "better" for social welfare, since we have already seen that the competitive extraction path maximises social welfare.

4 Costly extraction

We now relax the assumption of costless extraction and let firms face a (short run) cost function

$$c(t) \equiv c(q(t), x(t)),$$

which, notice, specifies that costs may depend not only on the quantity extracted q but also on the size of the stock x. Intuitively, extraction could get more costly as the size of a reserve diminishes (perhaps because of the need to access deeper or more sparse deposits, reduced oil or gas pressures, etc.).

For a competitive firm, the two key rules (3) and (4) can now be written (dropping the time arguments for convenience) as

$$p - c_q = \lambda$$

and

$$\dot{\lambda} = \lambda r + c_x,$$

where $c_q > 0$ and $c_x \leq 0$ are the first derivatives of the cost function with respect to q and x. Alternatively, if we write the firm's profit function as

$$\pi(t) \equiv p(t)\, q(t) - c(q(t), x(t)),$$

we could express these rules a little more neatly as

$$\pi_q = \lambda \tag{6}$$

and

$$\dot{\lambda} = \lambda r - \pi_x, \tag{7}$$

noting that $\pi_x = -c_x \geq 0$.

By substitution for λ from (6) into (7) and rearranging, Hotelling's Rule for a competitive firm now becomes

$$\frac{\dot{\pi}_q}{\pi_q} = r - \frac{\pi_x}{\pi_q}.$$

The most significant difference is the appearance of the final term on the left hand side of the expression. The extraction plan now has to take into account the effect upon extraction costs of reducing the size of the resource. If this effect is zero, so that $\pi_x = 0$, the rule is essentially the same as before, except that it is now expressed in terms of marginal profit rather than simply price. In effect, marginal profit here represents the *net price* of the resource once it is extracted. If, on the other hand, $\pi_x > 0$ (i.e., extraction costs are increased as the stock size is reduced), this implies that

$$\frac{\dot{\pi}_q}{\pi_q} < r,$$

that is, the optimal rate of increase of marginal profit is now *less than* the interest rate.

It is difficult to say much more in general terms about the efficient extraction path, which will now depend to a great extent on how the cost function is specified, in particular whether costs are reserve-dependent. Nevertheless, we would expect the inclusion of extraction costs to moderate the rate of price rise, and hence the rate at which the quantity extracted declines. If we assume no reserve-dependent costs, we can write Hotelling's Rule as

$$\frac{\dot{\pi}_q}{\pi_q} \equiv \frac{\dot{p} - \dot{c}_q}{p - c_q} = r.$$

Rearranging terms we can then find

$$\frac{\dot{p}}{p} = r\left[\frac{p - c_q}{p}\right] + \frac{\dot{c}_q}{p} < r,$$

which must be the case provided that $[p - c_q]/p < 1$ (assured as long as $q > 0$) and $\dot{c}_q \leq 0$ (which will be the case for most cost functions as long as the quantity extracted is non-increasing over time). With extraction costs, therefore, the extraction path becomes more gradual.

It is also difficult to be specific about the end point of the extraction path with costs. Exhaustion of the resource, for example, may not be optimal if costs are increasing as the size of the stock (reserve) decreases. The end price $p(T)$ may be less than the backstop price since, roughly speaking, extraction continues until either profits are reduced to zero or the resource is exhausted. The cost function may determine that extraction becomes unprofitable at a price at which demand is still positive.

5 Further reading

Look at Conrad, pp.77-91, and HSW, pp.216-232.