INTRODUCTION TO
NATURAL RESOURCE ECONOMICS

Lecture 4

Renewable resource exploitation: the fishery

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1 Introduction

This lecture extends the theory of resource exploitation to renewable natural resources, i.e., resources which exhibit *growth*. The classic (and most studied) example of a renewable natural resource is the capture fishery. Here we consider optimal exploitation of the fishery and the problem of "overfishing". In the next lecture we will look at the regulation or *management* of fisheries in order to prevent overfishing and move the fishery closer to an optimal pattern of exploitation.

2 A basic (dynamic) fishery model

Consider a very simple fishery in which there is just one (homogenous) stock owned by a single firm (a *sole owner*). We can define an (instantaneous) profit function, as before, as

$$\pi\left(q\left(t\right), x\left(t\right)\right) \equiv pq\left(t\right) - c\left(q\left(t\right), x\left(t\right)\right),$$

where $x\left(t\right)$ is the stock size and $q\left(t\right)$ is the harvest.

The resource owner's problem is then to maximise the present value of exploiting the resource, which we can write as

$$\max_q \int_0^T \pi(\centerdot)\, e^{-rt} dt,$$
$$\text{s.t.} \quad \dot{x} = g(x) - q(t),$$

where the constraint is the so-called *state equation*, which now explicitly includes a growth function $g(x)$.

The present value Hamiltonian for the problem is

$$\mathcal{H}(\centerdot) \equiv \pi(\centerdot)\, e^{-rt} + \mu(t)\,[g(x) - q(t)],$$

while its current value equivalent is

$$\mathcal{H}^c(\centerdot) \equiv \pi(\centerdot) + \lambda(t)\,[g(x) - q(t)].$$

The first order necessary conditions for an optimal solution can be summarised as

$$\mathcal{H}_q = 0 \quad \Rightarrow \quad \pi_q e^{-rt} = \mu,$$
$$\dot{\mu} = -\mathcal{H}_x \quad \Rightarrow \quad \dot{\mu} = -\pi_x e^{-rt} - \mu g'(x)$$

and

$$\dot{x} = \mathcal{H}_\mu \quad \Rightarrow \quad \dot{x} = g(x) - q(t),$$

in the case of the present value Hamiltonian, or

$$\mathcal{H}^c_q = 0 \quad \Rightarrow \quad \pi_q = \lambda,$$
$$\dot{\lambda} - r\lambda = -\mathcal{H}^c_x \quad \Rightarrow \quad \dot{\lambda} - \lambda r = -\pi_x - \lambda g'(x)$$

and

$$\dot{x} = \mathcal{H}^c_\lambda \quad \Rightarrow \quad \dot{x} = g(x) - q(t),$$

in the case of the current value Hamiltonian.

Notice that the conditions for q^* are essentially the same, except that $\mathcal{H}_q = 0$ is in present value terms, while $\mathcal{H}^c_q = 0$ is in current value terms. Also, as with any Lagrangian, taking a derivative with respect to the multiplier (shadow price) just returns the constraint, in this case the state equation. Thus we have $\dot{x} = \mathcal{H}_\mu = \mathcal{H}^c_\lambda = g(x) - q(t)$. The conditions for x^* are a bit different however, when we move from present value to current value: we have $\dot{\mu} = -\mathcal{H}_x$ but $\dot{\lambda} - r\lambda = -\mathcal{H}^c_x$ (in the previous lecture we saw how the current value condition could be derived from the present value condition). You should be able to see, though, that if $r = 0$ the conditions will be the same.

Let's just focus on the current value conditions

$$\pi_q = \lambda$$

and

$$\dot{\lambda} = \lambda r - \pi_x - \lambda g'(x) \quad \Rightarrow \quad \frac{\dot{\lambda}}{\lambda} = r - \frac{\pi_x}{\lambda} - g'(x).$$

These should, by now, be fairly familiar, except for the inclusion of the marginal growth rate $g'(x)$ in the condition for the evolution of the shadow price. These conditions state that, for an optimal exploitation path, current marginal profits should be equal to the shadow price of the stock, while the shadow price should be changing at a rate which depends upon (1) the interest rate, (2) the relative impact on profits of a change in the stock size, and (3) the marginal growth rate of the resource.

3 Sustainable fishery exploitation

When we were dealing with non-renewable resources, it didn't really make any sense to talk about *sustainable* exploitation. If a resource doesn't exhibit growth, then extracting the resource inevitably depletes it. With renewable resources, however, it should be possible to exploit the resource in such a way that exploitation is sustainable (indefinitely). This would, at least in principle, appear to be the best way of exploiting any renewable resource.

Let's assume that our time horizon T extends a long way into the future, if not to $T = \infty$. Let's also assume that we have got the fishery into a sustainable position or *steady state*, where harvest equals growth and hence the stock size is neither increasing nor decreasing. A steady state therefore implies

$$\dot{x} = g(x) - q(t) = 0$$

and hence

$$q(t) = g(x).$$

It also implies that $\dot{\lambda} = 0$ and hence

$$r - \frac{\pi_x}{\lambda} - g'(x) = 0.$$

If we substitute $\pi_q = \lambda$ into this expression, we get

$$r - g'(x) - \frac{\pi_x}{\pi_q} = 0,$$

or, by rearranging,

$$r = g'(x) + \frac{\pi_x}{\pi_q}. \tag{1}$$

Expression (1) is sometimes known as the **fundamental equation of renewable resources**. We will return to this later.

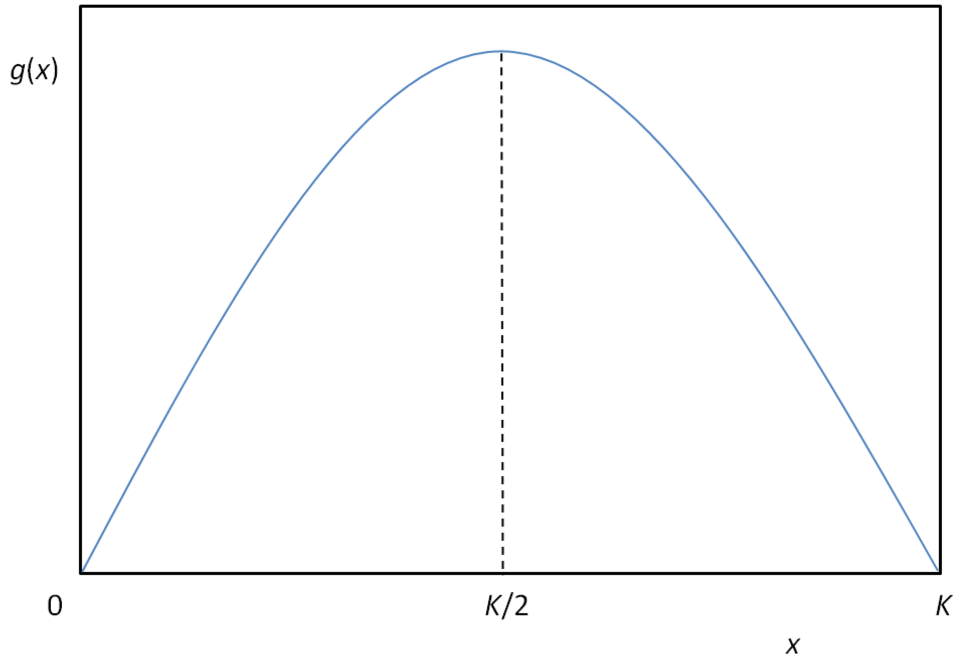
4 The growth function

We have so far expressed the growth function in very general terms as $g(x)$, which we have implicitly assumed to be positive for at least some values of x. There are many specific growth models and these can be very complex if they take into account, for example, random fluctuations, interactions between different species of fish, migration, or an age-structured population.

The simplest growth model, and one which is often used in simple bio-economic models, is the *logistic* growth function. This can be written as

$$g(x) \equiv \gamma x \left[1 - \frac{x}{K}\right],$$

where γ is the *intrinsic growth rate* of the resource and K is the environmental *carrying capacity.* The parameter K represents the maximum stock size that can be accommodated by the environment in which the fish live, because of space or food constraints, for example. A (static) plot of $g(x)$ against x yields a dome-shaped curve where $g(x) = 0$ at $x = 0$ and $x = K$ and $g(x)$ has a maximum where $g'(x) = 0$ and hence $x = K/2$.



In the absence of any harvesting, in this simple deterministic (non-random) model, the equilibrium fish stock will be equal to K. If we reduce the stock to $K/2$, however, we can obtain a sustainable (physical) yield equal to the maximum value of $g(x)$. This is referred to as the *maximum sustainable yield* or MSY. Note that we have assumed that growth is positive for any positive stock size, i.e., implicitly, there is no *minimum viable population*. It is quite possible that there would be a minimum viable population, in which case $g(x)$ would become negative below a certain x (and the stock would collapse to zero).

This model of sustainable yield based upon the logistic growth model is known as a *surplus production model*, since it models yield (potential harvest) as a surplus which can be taken while maintaining the stock at a given level.

MSY is sometimes advocated as an appropriate goal for fisheries management, but there are problems with the concept of MSY. Firstly, from a purely biological perspective, MSY only really makes sense in the context of a simple deterministic model. In the real world, fish stocks fluctuate considerably due to environmental influences and random behaviour and MSY is therefore variable and hard to define. Also, different commercially important fish stocks frequently coexist and are caught together. Since these stocks often either compete for food or provide food for each other, it may be impossible to define MSY simultaneously for different stocks in a multi-species fishery.

Secondly, from an economic perspective, setting MSY as a target takes no account of the *costs* of catching fish.

5 The static model of fishery exploitation

We can adapt the logistic stock growth model into a simple fishery model as follows. To begin with, let harvest equal sustainable yield. Then, assume that we have some reliable measure of fishing inputs or "effort" (say, the number of vessels) and for simplicity let the "catch per unit of effort" (CPUE) be a constant proportion of the stock. Formally, we assume that harvest (catch) as a function of fishing effort can be written

$$q(e) \equiv \theta e x,$$

where e is effort and θ is a "catchability coefficient". If $q(e) = g(x)$, then

$$\theta e x = \gamma x \left[1 - \frac{x}{K}\right]$$

and hence

$$x = K \left[1 - \frac{\theta e}{\gamma}\right],$$

so that we can express the harvest as a function of effort as

$$q(e) = \theta e K \left[1 - \frac{\theta e}{\gamma}\right].$$

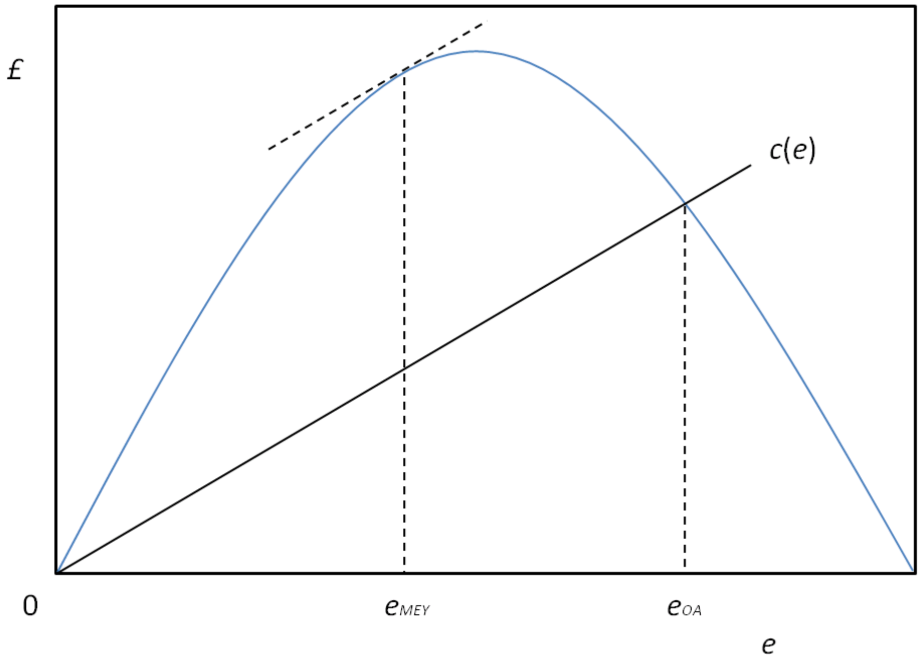
Note that the relationship between fishing effort and stock size is implicitly a negative one.

Next, assuming that the price of fish is unaffected by the size of the harvest (perhaps because many other fisheries supply the same product) we can simply multiply the harvest by the price to obtain a graph of (sustainable) revenue against effort. Then profits in the fishery are given by

$$pq(e) - c(e),$$

where $c(e)$ is the cost of effort.[1] Profits are clearly maximised where $pq'(e) = c'(e)$ and a sole owner, would, we assume operate with this optimal level of effort, which we can label as e_{MEY} since the corresponding yield is often referred to as the *maximum economic yield* or MEY. In the graph, the difference between revenues and costs at e_{MEY} is an economic surplus which can be interpreted as the *resource rent*.

[1] Note that $c(e)$ is assumed to include the economic costs of *all* inputs, including capital.



Sole ownership of a fishery is very unusual, however. In most fisheries a large number of competitive firms are involved and effort will, if permitted, tend to be attracted into the fishery until economic profits are reduced to zero, a level of effort which we can refer to as the *open access* level e_{OA}. In the graph we have drawn the effort cost curve in such a way that e_{OA} is much greater than e_{MEY} (which implies a much smaller stock size), but this would not necessarily be the case. The important point is that the competitive outcome, where economic profits are zero, may be optimal in other industries but is sub-optimal in a fishery since all the resource rent has been *dissipated* through excessive investment in harvesting capacity (fishing effort). This is an example of *market failure*, which can here be attributed to the lack of a price on the fish stock itself. In the case represented in the graph, we also have a relatively depleted stock and reduced harvests, but remember that, in this model, all points on the revenue curve are implicitly *sustainable*.

This fishery model (called the Gordon-Schaefer model) is commonly used to illustrate the problem of (economic) overfishing in an open access (i.e., unregulated) fishery, but remember that it is a static (and long run) model: no account is taken of the discount rate (which is, implicitly, zero). Nevertheless, it is useful since it highlights the need for a social planner to regulate or *manage* the fishery in order to maximise its economic value (and prevent stock depletion).

6 The dynamic optimum

We return to the so-called fundamental equation of renewable resources, which we obtained previously. This was expressed in terms of harvest q as

$$r = g'(x) + \frac{\pi_x}{\pi_q},$$

but we could easily derive an equivalent expression in terms of fishing effort.

In either case, the fundamental equation states that, to be exploiting the fish stock sustainably and optimally, we need to equate the discount rate r with the marginal growth rate of the stock $g'(x)$ *plus* the marginal value of the stock in relation to the marginal value of harvest (or effort). This ratio is often referred to as the *marginal stock effect* (MSE).

If the marginal value of the stock is zero (which could, conceivably, be the case for strongly schooling or shoaling species) then the MSE is equal to zero and we would simply have

$$r = g'(x),$$

i.e., the discount (interest) rate is equated with the marginal growth rate. For a positive discount rate, this implies that the optimal stock size would correspond to the upward-sloping portion of the sustainable yield curve, *below* the stock size associated with MSY. Indeed, with a zero MSE, MSY would only be optimal in the case of a zero discount rate.

If the MSE is positive, this will determine an optimal stock size that is larger, all else equal. If the MSE exceeds the discount rate, then the optimal stock size could be where $g'(x)$ is negative, i.e., on the downward-sloping portion of the yield curve.

It should be apparent that with a small MSE and a very high discount rate, the optimal solution might be to rapidly fish the stock down to a very low level (where the marginal growth rate is highest). This is equivalent to disinvesting in the resource. In the case of a very slow growing species, so that $r > g'(x)$ everywhere, it might be optimal to deplete the resource entirely. This is a theoretical possibility for a sole owner, although not, perhaps, for a social planner who would (we assume) value the resource for its existence, *per se*.

The social planner (or sole owner) takes into account the dynamics of the stock and hence the rule for optimal harvest takes into account the shadow price of the stock. A single competitive firm participating in an open access (and unregulated) fishery, on the other hand, has no incentive to take account of the stock constraint (why?) and therefore continues to harvest until the marginal profit from harvesting is zero. This implies excessive levels of harvest and hence a sub-optimal outcome. If we rearrange the fundamental equation to

$$\pi_q = \frac{\pi_x}{r - g'(x)}$$

we can see that, for a sole owner, $\pi_q = 0$ is implied by a discount rate of $r \to +\infty$. In effect, individual firms in an open access fishery behave as if they have an infinite discount rate. To change this behaviour usually requires some sort of regulation or management of the fishery. We will look at this in the final lecture.

7 Further reading

Look at HSW, pp.266-283. Conrad, pp.9-16 and pp.32-49, covers similar material from a discrete time perspective.