Non-renewable resource exploitation:
externalities, exploration, scarcity and rents
NRE - Lecture 3

Aaron Hatcher

Department of Economics
University of Portsmouth
Externalities
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- External (social) costs
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- Example: pollution
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- Effects not covered by markets and so difficult to value
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- $MB = MC$
Externalities

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- Example: pollution
- Effects not covered by markets and so difficult to value
- Trade off between costs and benefits
- $MB = MC$
- Optimal pollution does not imply a fair distribution of costs and benefits!
Externalities contd.

- A *social planner* would want a firm to maximise

\[
\int_0^T \left[ pq - c(q, x) - d(z, a) - \theta(a) \right] e^{-rt} \, dt
\]
Externalities *contd.*

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- Here \( z \equiv \alpha q \) is a pollutant generated by extraction; \( d(\cdot) \) is a *damage function* and \( \theta(a) \) are *abatement* costs
Externalities *contd*.

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- Normalise \( \alpha \) to 1 so that \( z = q \)
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Optimality now requires

\[
p - c_q = \lambda + d_z, \quad \theta_a = -d_a
\]
Externalities *contd.*

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- Optimality now requires

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\]

- If \( d(\cdot) \equiv d(z - a) \) then \( d_z = -d_a \) and we can write

\[
p - c_q - \theta_a = \lambda
\]

for a “socially responsible” firm
Firms must have an *incentive* to incur abatement costs.
Externalities *contd.*

- Firms must have an *incentive* to incur abatement costs
- How?
Externalities *contd.*

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  - pollution taxes
Externalities *contd.*

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- How?
  - pollution taxes
  - standards
  - tradeable permits
  - backed up by enforcement (inspections and penalties)
- Note: there may be stock as well as flow effects
Externalities contd.

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- Firms forced to internalise the (social) costs of pollution
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**How?**
- pollution taxes
- standards
- tradeable permits

**Backed up by enforcement (inspections and penalties)**
**Firms forced to internalise the (social) costs of pollution**

**Note:** there may be *stock* as well as *flow* effects
Exploration

- Few general conclusions
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- New discoveries may not be developed immediately, depending on extraction costs
Exploration

- Few general conclusions
- Costly and subject to uncertainty
- New discoveries may not be developed immediately, depending on extraction costs
- Models of exploration are complex!
Scarcity
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- An *economic* definition of scarcity

\[ \dot{p} = \dot{\lambda} + \dot{c} q \]

Clearly, \( \dot{p} \) and \( \dot{\lambda} \) can have opposite signs.
Scarcity

- An *economic* definition of scarcity
- Must depend on *demand* (price) and extraction *costs*
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- The *shadow price* \( \lambda \) (the difference between price and marginal cost) is regarded as the correct measure of economic scarcity
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- The *shadow price* \( \lambda \) (the difference between price and marginal cost) is regarded as the correct measure of economic scarcity
- A *trend* in the shadow price is the best indicator of resource scarcity
An economic definition of scarcity

Must depend on demand (price) and extraction costs

A resource becomes more scarce if demand (price) increases and less scarce if demand decreases

Economic scarcity is decreasing if extraction costs increase and prices don’t

The shadow price $\lambda$ (the difference between price and marginal cost) is regarded as the correct measure of economic scarcity

A trend in the shadow price is the best indicator of resource scarcity

Since

$$\dot{p} = \dot{\lambda} + \dot{c}_q,$$

clearly $\dot{p}$ and $\dot{\lambda}$ can have opposite signs
Rent capture
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- Common charges are taxes on revenues/quantities (*severance* tax) or profits (*royalty*)
Rent capture

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- Common charges are taxes on revenues/quantities (severance tax) or profits (royalty)
- With a revenue tax \( \tau \) the firm maximises

\[
\int_0^T \left[(p - \tau) q - c(q, x)\right] e^{-rt} \, dt
\]
Rent capture

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- Government may try to \textit{capture} (some) rent for society’s benefit
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- Effect similar to an increase in the costs of extraction: extraction ceases earlier and more resource is left in the ground
Rent capture

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- With a revenue tax $\tau$ the firm maximises
  \[\int_0^T \left[ (p - \tau)q - c(q, x) \right] e^{-rt} dt\]
  - Effect similar to an increase in the costs of extraction: extraction ceases earlier and more resource is left in the ground
- A revenue tax is distortionary
Rent capture *contd*.

- With a profits tax $\tau$ the firm's objective function is

$$\int_0^T [1 - \tau] [pq - c(q, x)] e^{-rt} \, dt$$
Rent capture *contd.*

- With a profits tax $\tau$ the firm’s objective function is
  \[ \int_0^T [1 - \tau] [pq - c(q, x)] e^{-rt} dt \]

- The tax does not change profit-maximising behaviour and hence is not distortionary
Rent capture *contd.*

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Rent capture *contd.*

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- But, a profits tax reduces the *present value* of the resource
- Government may (also) sell extraction or prospecting rights
Rent capture *contd*.

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  \[
  \int_0^T [1 - \tau] [pq - c(q, x)] e^{-rt} dt
  \]
  - The tax does not change profit-maximising behaviour and hence is not distortionary.
  - But, a profits tax reduces the *present value* of the resource.
  - Government may (also) sell extraction or prospecting rights.
  - Or, in some cases, the resource itself.
Dynamic optimisation: the Hamiltonian
Dynamic optimisation: the Hamiltonian

- Define an instantaneous profit function

\[ \pi(q(t), x(t)) \equiv pq(t) - c(q(t), x(t)) \]
Define an instantaneous profit function

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The (social planner’s) problem is to

\[
\max_q \int_0^T \pi(\cdot) e^{-rt} dt \\
\text{s.t. } \dot{x} = f(x(t), q(t)), \quad x(0) = x_0
\]
Define an instantaneous profit function

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The (social planner’s) problem is to

\[ \max_q \int_0^T \pi (\dot{\cdot}) e^{-rt} dt \]

s.t.  \( \dot{x} = f(x(t), q(t)) \),  \( x(0) = x_0 \)

Define a Lagrangian function

\[ \mathcal{L} \equiv \int_0^T \left\{ \pi (\dot{\cdot}) e^{-rt} + \mu(t) [f(\dot{\cdot}) - \dot{x}] \right\} dt \]

where

\[ \mu(t) \equiv \lambda(t) e^{-rt} \]
Define a (present value) Hamiltonian

\[ H(\cdot) \equiv \pi(\cdot) e^{-rt} + \mu(t) f(\cdot) \]

so that

\[
\mathcal{L} = \int_{0}^{T} H(q, x, \mu, t) \, dt - \int_{0}^{T} \mu(t) \dot{x} \, dt
\]
Define a (present value) *Hamiltonian*

\[ H(\cdot) \equiv \pi(\cdot) e^{-rt} + \mu(t) f(\cdot) \]

so that

\[ \mathcal{L} = \int_{0}^{T} H(q, x, \mu, t) \, dt - \int_{0}^{T} \mu(t) \dot{x} \, dt \]

Integrate the final term *by parts* to obtain

\[ \int_{0}^{T} \mu(t) \dot{x} \, dt = \mu(T) x(T) - \mu(0) x_0 - \int_{0}^{T} \dot{\mu} x(t) \, dt \]
Dynamic optimisation contd.

- Rewrite the Lagrangian as

\[
\mathcal{L} \equiv \int_0^T \{ \mathcal{H}(\cdot) + \dot{\mu}x \} \, dt + \mu(0)x_0 - \mu(T)x(T)
\]
Dynamic optimisation contd.

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- Differentiate w.r.t. \( q \) and \( x \) to get the FOCs

\[
\mathcal{L}_q = \mathcal{H}_q = \pi_q e^{-rt} + \mu f_q = 0
\]

\[
\mathcal{L}_x = \mathcal{H}_x + \dot{\mu} = \pi_x e^{-rt} + \mu f_x + \dot{\mu} = 0
\]
Dynamic optimisation *contd.*

- Rewrite the Lagrangian as

\[ \mathcal{L} \equiv \int_{0}^{T} \{ \mathcal{H}(\dot{x}) + \dot{\mu}x \} \, dt + \mu(0)x_0 - \mu(T)x(T) \]

- Differentiate w.r.t. \( q \) and \( x \) to get the FOCs

\[
\begin{align*}
\mathcal{L}_q &= \mathcal{H}_q = \pi_q e^{-rt} + \mu f_q = 0 \\
\mathcal{L}_x &= \mathcal{H}_x + \dot{\mu} = \pi_x e^{-rt} + \mu f_x + \dot{\mu} = 0
\end{align*}
\]

- Hence we require

\[
\pi_q e^{-rt} = -\mu f_q \quad \Rightarrow \quad \pi_q e^{-rt} = \mu
\]

given \( f(\cdot) \equiv g(x) - q \) and hence \( f_q = -1 \)
Dynamic optimisation \textit{contd.}.

- Rewrite the Lagrangian as

\[
\mathcal{L} \equiv \int_{0}^{T} \{\mathcal{H}(\dot{s}) + \dot{\mu}x\} \, dt + \mu(0)x_0 - \mu(T)x(T)
\]

- Differentiate w.r.t. \(q\) and \(x\) to get the FOCs

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\mathcal{L}_q = \mathcal{H}_q = \pi_q e^{-rt} + \mu f_q = 0 \\
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given \(f(\dot{s}) \equiv g(x) - q\) and hence \(f_q = -1\)

- This is equivalent to the familiar \textit{current period} condition

\[
\pi_q = \lambda
\]
Dynamic optimisation *contd.*

- We also require

\[ \dot{\mu} = -\pi_x e^{-rt} - \mu f_x \]
Dynamic optimisation *contd.*

- We also require
  \[
  \dot{\mu} = -\pi x e^{-rt} - \mu f_x
  \]
- From \( \mu(t) \equiv \lambda(t) e^{-rt} \) we can find
  \[
  \dot{\mu} = [\dot{\lambda} - r\lambda] e^{-rt}
  \]
Dynamic optimisation *contd*.

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- Hence, in current value terms, we have
  \[
  \dot{\lambda} = -\pi_x - \lambda [f_x - r]
  \]
Dynamic optimisation *contd.*

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- For a non-renewable resource, \( f_x = g'(x) = 0 \) and so
  \[ \dot{\lambda} = \lambda r - \pi_x \]
Dynamic optimisation *contd.*

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  \[ \dot{\lambda} = -\pi x - \lambda [f_x - r] \]

- For a non-renewable resource, \( f_x = g'(x) = 0 \) and so
  \[ \dot{\lambda} = \lambda r - \pi x \]

- Or, without resource-dependent costs,
  \[ \dot{\lambda} = \lambda r \]
Dynamic optimisation *contd.*

- What does the Hamiltonian represent?

\[
H(t) = \pi(t) e^{rt} + \mu(t) f(t)
\]

Or, in current value terms

\[
H_c(t) = \pi(t) + \lambda(t) f(t)
\]

The Hamiltonian measures the total rate of increase in the value of the resource. \( \pi(t) \) is the net flow of returns from the resource, and \( \lambda(t) f(t) \) is the increase in the value of the stock.
What does the Hamiltonian represent?

The present value Hamiltonian is

\[ \mathcal{H}(t) \equiv \pi(t) e^{-rt} + \mu(t) f(t) \]

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The Hamiltonian measures the total rate of increase in the *value* of the resource.
Dynamic optimisation *contd.*

- What does the Hamiltonian represent?
- The present value Hamiltonian is
  \[ H(\cdot) \equiv \pi(\cdot) e^{-rt} + \mu(t) f(\cdot) \]
- Or, in current value terms
  \[ H^c(\cdot) \equiv \pi(\cdot) + \lambda(t) f(\cdot) \]
- The Hamiltonian measures the total rate of increase in the *value* of the resource
  - \( \pi(\cdot) \) is the net flow of returns from the resource
What does the Hamiltonian represent?
The present value Hamiltonian is
\[ \mathcal{H}(s) \equiv \pi(s) e^{-rt} + \mu(t) f(s) \]

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\[ \mathcal{H}^c(s) \equiv \pi(s) + \lambda(t) f(s) \]

The Hamiltonian measures the total rate of increase in the value of the resource
- \( \pi(s) \) is the net flow of returns from the resource
- \( \lambda(t) f(s) \) is the increase in the value of the stock