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Answer 5 out of the following 6 questions (20% each). There is also a bonus. If you have time, then you can answer the 6th question and I will take the 5 **best** questions for your grade. Time 2:30 hours with 30 minute extension.

1 Price Discrimination.

Quick Photo shop is able to easily print digital photos in a 1/2 hour. However, they offer two options: (a) to receive your photos in 1/2 an hour or (b) to receive your photos in 3 hours. Assume there are two representative customers to the store: Jim and Sean. Jim values receiving his photos in 1/2 hour at 20 argurot and in 3 hours at 14 argurot. Sean values receiving his photos in 1/2 an hour at 14 argurot and 3 hours at 12 argurot. Assume it costs the shop 5 argurot to develop a photo. Assume the shop can charge a different price for each service but cannot charge Jim and Sean a different price.

(i) Assuming that Jim and Sean develop the same number of photos, what prices should the shop charge?

(ii) Let us say that Jim and Sean have a different number of photos to develop. At what ratios of Jim's photos to Sean's photos would the shop only sell the 1/2 hour option and at what prices. (Hint: there are two possible prices.)

Answer.

The shop's best options are to

(a) sell photos to only Jim in 1/2 an hour,

(b) sell photos to both Jim and Sean for 1/2 an hour, or

(c) sell photos to Jim for 1/2 an hour and Sean for 3 hours.

(i) if both Jim and Sean both want to develop n pictures, then for option (a), the photo shop can make $(20-5)*n = 15n$ charging 20 argorot. For option (b) (14-5)*2*n=18n, charging 14 argorot. The store with option (c) can make $(18-5)*n + (12-5)*n = 20n$ charging 12 argorot for 3 hours and 18 argorot for 1/2 an hour. Option (c) is best.

(ii) Denote n_j and n_s as the number of photos Jim and Sean want to develop. For option (a), the photo shop can make $(20-5)*n_j = 15n_j$ charging 20 argorot. For option (b) $(14-5)*(n_j+n_s) = 9(n_j+n_s)$, charging 14 argorot. The store with option (c) can make $(18-5)*n_j + (12-5)*n_s = 13n_j + 7n_s$ charging 12 argorot for 3 hours and 18 argorot for 1/2 an hour. Option (a) is best if $15n_j > 13n_j + 7n_s$ or $n_j/n_s > 7/2$. Option (b) is best if $9(n_j+n_s) > 13n_j + 7n_s$ or $2n_s > 4n_j$ or $n_j/n_s < 1/2$. Note if (a) is better than (c) then it is also better than (b). Also if (b) is better than (c), then it is also better than (a).

2 Bank Runs.

Take the Diamond-Dybvig model described in class with 2 impatient depositors and 2 patient depositors.. Each depositor invested £1000 in the bank and was offered a contract: withdrawing today pays £1000, withdrawing tomorrow pays

£2000 ($R = 2.0$). The bank had two possible means of investing its money: a long-term investment and a short-term investment. The long-term investment pays $R = 2.0$ times the amount invested tomorrow. Early liquidation of the investment today pays $L = 0.5$ times the original amount invested. The short term investment pays the original amount if it is withdrawn today or tomorrow.

(i) Assuming that the patient depositors wait until tomorrow to withdraw and the impatient depositors withdraw today, how should the bank divide its assets between the short-term and long-term investment to match demands?

(ii) Assuming that all the impatient depositors withdraw their money today, represent the decisions of the patient depositors as 2x2 game (draw it).

(iii) Indicate any pure-strategy Nash equilibrium of the game .

(iv) Assume that L is no longer .5, for what values of L ($0 < L < 1$) would we have the same equilibria as in (iii)?

Answer.

(i) The bank needs to put £2000 in the short-term investment and £2000 in the long-term investment. This way it can pay the 2 impatient depositors £1000 each and the 2 patient depositors £2000 each.

(ii) If all 4 depositors withdraw today, the bank will only have 2000+2000*.5=300 or £750 each.

If 3 depositors withdraw today (2 impatient + 1 patient), then the bank will be able to pay demands today, but will have to liquidate all of its illiquid assets. This leaves nothing for tomorrow. Thus, the 2x2 game can be written as that in Figure 1.

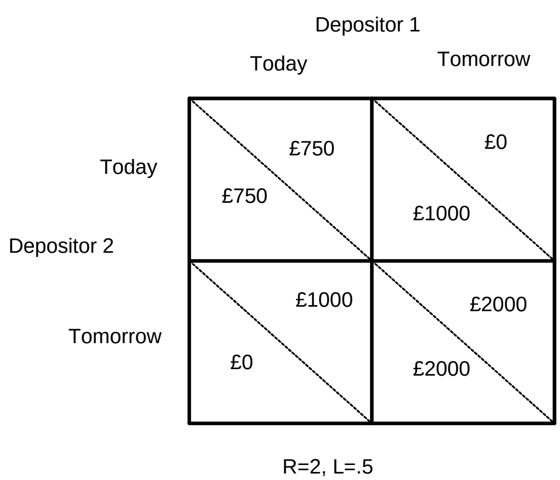

Figure 1: Game between the two patient depositors.

(iii) There are two pure-strategy N.E.: both withdraw today and both withdraw tomorrow. In both cases, neither has incentive to change their strategy.

(iv) Remember to give the patient depositor 1000 today, the bank must liquidate an amount x where 1000=L*x. Note when L=.5, X must be 2000. Solving gives us x=1000/L. The remaining amount is (2000-1000/L). This amount is 0 when L<=.5 since it can't be negative. The amount the remaining patient depositor would receive is then R(2000-1000/L). There are the two equilibria in (iii) if this amount is LESS than the amount the patient depositor would get if all withdrew today. That amount is (2000+2000*L)/4. Thus, there are two equilibria if

$2(2000 - 1000/L) \leq (2000 + 2000 * L)/4$

Simplifying we have that there are two equilibria if $L^2 - 7L + 4 >= 0$ or $L < 0.63$. For example, take L=.8. If both patient depositors withdraw today, then they would get on average 900. If one decides to wait until tomorrow and the other withdraws today, the one withdrawing tomorrow gets 2(2000-1000/.8)=1500. Thus, both withdrawing today can't be an equilibrium. Alternatively, if L=.6, both withdrawing today yields 800. One waiting for tomorrow would receive 2(2000-1000/.6)=666. Intuitively, if L is higher, there is less early liquidation costs. So the bank would have more money left over to pay the patient depositors what they are owed.

3 Signalling.

When a gazelle is approached by a predator, it sometimes jumps high in the air rather than running away. Assume there are two types of gazelles, strong and weak. Only strong gazelles can jump. Strong gazelles that run and are chased by the predator have a 10% chance of being caught. Strong gazelles that jump and then run have a 20% chance of being caught. Weak Gazelles have a 50% chance of being caught. A predator has a benefit of 100 utils for eating a gazelle, 0 for not running and not catching a gazelle and a utility of -30 for running and not catching a gazelle.

(i) For what fraction of strong gazelles is the only equilibrium is where we get that strong gazelles always jump?

(ii) What is the behaviour of the predators in such an equilibrium?

Answer:

The payoff of the predator going after a strong gazelle that has jumped is $.2 * 100 + .8(-30) = -4$

The payoff of the predator going after a strong gazelle that has not jumped is $.1 * 100 + .9(-30) = -17$

The payoff of the predator going after a weak gazelle that has not jumped is $.5 * 100 + .5(-30) = 35$.

If f is greater than f^* where $-17 \cdot f^* + (1 - f^*) \cdot 35 = 0$ then there can be a pooling equilibria where the predator ignores them all. Solving yields $f^* = 35/52 = 0.673$.

(ii) In the equilibrium, where strong gazelles always jump, the predator ignores all gazelles that jump and chases after all that don't jump. The predators ignore the jumping gazelles, because they know they are strong and will have a

Payoffs: Gazelle, Predator

	Chase	Ignore
Jump (strong)	**80, -4**	**100, 0**
Don't jump (strong)	**90, -17**	**100, 0**
Don't jump (weak)	**50, 35**	**100, 0**

payoff of -4 going after them. In this equilibrium, all the gazelles that simply run are weak. It is worthwhile for the predator to chase them since the expected payoff is then 35.

4 Vertical Markets.

Pineapple is a company with a hot product called the fruit camera (the f-camera for short). It is a camera that automatically analyzes and categorizes pictures by both place and people in them. Currently, Pineapple is selling its camera only in its pineapple stores. The marginal cost of each camera is \$50 and it faces a demand of $q = 250 - p$.

(i) What price should Pineapple charge consumers to maximize profits?

Pineapple now is considering getting out of the retailing business and selling through retailers. The retailers buy the phone and sell it to the public.

(ii) What is the equilibrium price that Pineapple will charge the retailer and the price the retailer will charge the public?

Answer:

(i) profit is (250-p)(p-50). optimal price is p=150.

(ii) revenue is (250-q)*q, mr for retailer is 250-2q. Rev. for the Pineapple is (250-2q)q. MR for pineapple is 250-4q.

since $MC = 50$, we have $q = 200/4 = 50$ so 150 is the price to the retailer. The retailor then sets $250 - 2q = 150$ or $q = 50$ for a price of 200.

5 Network Externalities.

People that watch the Euro 2008 enjoy the game partially upon how many others are watching. For instance, they can go to work the next day and can avoid work by chatting with colleagues about the games. Champs is a company that buys the rights to broadcast the games on a small Greek Island with a population of 1000. It is charging $p = 18$ euros for a package to watch all the games. These people are indexed by i from 1 to 1000. Person i has value $v_i = i$. Person i has utility (in euros) for seeing the matches of $\frac{vi \cdot n}{5000}$, where n is the total number on the island watching the games.

(i) If everyone believes $n = 500$, which people will purchase the games?

(ii) What is the threshold number of people buying the rights to see the game above which it will be a success and below which it will be a failure?

(iii) What is the equilibrium number of packages sold for it to be a success?

(iv) What is the equilibrium number of packages sold for it to be a failure?

(v) Assuming it will be a success and zero marginal cost, would Champs do better charging a price of 48?

Answer:

(i) If person i believes 500 watch the matches then i's utility for purchasing the game will be $p - \frac{vi \cdot n}{5000} = 18 - \frac{vi \cdot 500}{5000} = 18 - \frac{vi}{10}$. Thus, anyone with vi>180 will watch the game. This is 820 people.

(ii-iv) If the cutoff is v^* than $n^* = 1000 - v^*$ will watch the game. Hence, $v^* = 1000 - n^*$. The interior equilibria are determined by $18 = \frac{(1000-n^*)n^*}{5000}$ or $90000 = (1000 - n^*)n^*$. It is not too hard to see that the solutions are 100 and 900. The threshold is 100. The success is 900 and the failure is 0.

(v) Repeat the above with p=48. We have $48 = \frac{(1000-n^*)n^*}{5000}$ or $240000 = (1000 - n^*)n^*$ and solutions of 600 and 400. The profit here if it is a success is 600*48 and before it was 900*18. While the quantity sold drops, the overall profit goes up, so it is worthwhile to increase the price.

5.1 Bill and Ted's big split: Subgame perfection

Bill and Ted were best friends for 10 years. They were so close, that they shared a prized collection of English football cards. This consists of 4 cards of famous players: Rooney (R), Owen (O), Beckham (B), and Shearer (S). They got in a fight when one of them became a bigger star and decided to split up the collection. They decided to take turns choosing cards. First Bill chooses, then Ted, then Bill, then Ted. (Bill got to choose first since he won a skate board race.) Bill prefers R to O to B to S. Ted prefers O to B to S to R. Write the extensive form game of this.

(1) What will be the decisions if they choose myopically only according to their preferences (that is, they choose the most prefered of the cards left)?

(2) What will be the decisions if they act strategically and take into account the future moves (subgame perfection)?

Answer:

(1) Since Bill chooses first and chooses his preferred item, he will choose R. Then Ted will choose O. Then Bill will choose B and then Ted S.

(2) If there are choosing strategically, then you could write out the game tree and use backward induction. Bill knows that Ted will never choose R. Knowing this Bill can choose O first and then R second. Ted will choose either B first and then S or S first and then B. Note that Bill does better here.

Bonus (moed Bet).

Guess a number between 0 and 100. The closest person to the 33.333+(2/3 times the average number) wins a bonus of 1% on the final. Ties will be broken randomly.

Winning number: 75. The equilibrium here is 100.