Moed Aleph. University of Haifa.

Answer 5 out of the following 6 questions (20% each). There is also a bonus. If you have time, then you can answer the 6th question and I will take the 5 **best** questions for your grade. Time 2:30 hours with 30 minute extension.

1 Price Discrimination.

The university has decided that only special calculators can be used during the exam. The economics department has a monopoly on the calculators that are sold before and not during the exam. Both poor and rich students taking the micro exam. Poor students feel that they can get by with only one calculator. Rich students feel that a second calculator would be helpful in case the first calculator breaks down. The following table shows their valuations (note the number under the heading "2 calculators" means the valuation for two calculators rather than for the 2nd calculator). Assume it costs the university 8 pounds per calculator. (Assume if indifferent in valuation terms to buying or not, students buy.) Assume that half the students are rich and half are poor.

	1 calculator	2 calculators
poor	£20	£20
rich	£30	£40

(i) If the university could only charge one price per calculator, independent of who buys it or how many, what would they charge?
(ii) If the university could fully price discriminate (tell who is who and charge based on quantity), what would they charge for the various combinations?
(iii) If the university could not tell who is who, but can charge different prices for different quantities what would they charge?
(iv) If the university could tell who is who, but must charge a constant price per calculator, what would they charge?
(v) How do the profits compare in all the cases?
(vi) If one-third of students were poor and two-thirds rich, and the university could not tell who is who, but can charge different prices for different quantities what would they charge?

1.1 Answers (out of 20 points)

(i) 3 points. (no price discrimination) The uni will charge p=20 and sell one calculator to each student. The profit (per student) will be $\pi = .5 * (20 - 8) + .5 * (20 - 8) = 12$.

Note if the uni charged p=30, it would sell only one calculator to the rich students. $\pi = .5 * (30 - 8) = 11$.

(ii) 3 points. (full price discrimination) The uni will charge the poor students 20 for one calc. and more than 20 for two. For the rich students, the uni will charge 40 for two calculators and more than 30 for one. This way the uni will make $\pi = .5 * (40 - 16) + .5 * (20 - 8) = 18$. Note the university gains from selling two to the rich students since the gain in revenue is $40 - 30 = 10$ and they give up 8 in costs.

(iii) 3 points. (2nd degree price discrimination) The uni has to worry about a rich student pretending to be poor. Thus, it can only charge 10 more for 2nd calculator. Here the university will charge 20 for one calculator and 30 for two calculators. In this case, profits will be $\pi = .5 * (30 - 16) + .5 * (20 - 8) = 13$. (Note that the university will not profit by only trying to sell to the rich students since profits for selling two to the rich and none to the poor will be $\pi = .5 * (40 - 16) = 12$.

(iv) 3 points (3rd degree price discrimination). In this case, the uni can charge 20 per calc to the poor students and 30 per calc to the rich students. This yields profits of $\pi = .5 * (30 - 8) + .5 * (20 - 8) = 17$

(v) 4 points. We see the profits above. The order of profits is (ii)>(iv)>(iii)>(i)

(vi) 4 points. This was the tough part. $\pi = 2/3 * (30 - 16) + 1/3 * (20 - 8) = 12$. However, if the university focuses on just the rich students, it can charge 40 for two calculators and more than 30 for one, the profit would be $\pi = 2/3 * (40 - 16) = 16$.

2 Bank Runs.

Take the Diamond-Dybvig model described in class with 4 impatient depositors and 2 patient depositors. (NOTICE the odds of being impatient are now 2/3). Each depositor invested £1000 in the bank and was offered a contract: withdrawing today pays £1000, withdrawing tomorrow pays £1500 ($R = 1.5$). The bank had two possible means of investing its money: a long-term investment and a short-term investment. The long-term investment pays $R = 1.5$ times the amount invested tomorrow. Early liquidation of the investment today pays $L = 0.6$ times the original amount invested. The short term investment pays the original amount if it is withdrawn today or tomorrow.

(i) Assuming that the patient depositors wait until tomorrow to withdraw and the impatient depositors withdraw today, how should the bank divide its assets between the short-term and long-term investment to match demands?

(ii) Assuming that all the impatient depositors withdraw their money today, represent the decisions of the patient depositors as 2x2 game (draw it).

(iii) Indicate any pure-strategy Nash equilibrium of the game .

(iv) If one of the impatient depositors becomes unwell and decides to wait until tomorrow. This is known to the patient depositors. If both patient depositors wait until tomorrow, how much will the receive? Are there still the same equilibria as in (iii)?

2.1 Answers (out of 20 points)

(i) (5 points) The bank needs to put £4000 in the short-term investment and £2000 in the long-term investment. This way it can pay the 4 impatient depositors £1000 each and the 2 patient depositors £1500 each.

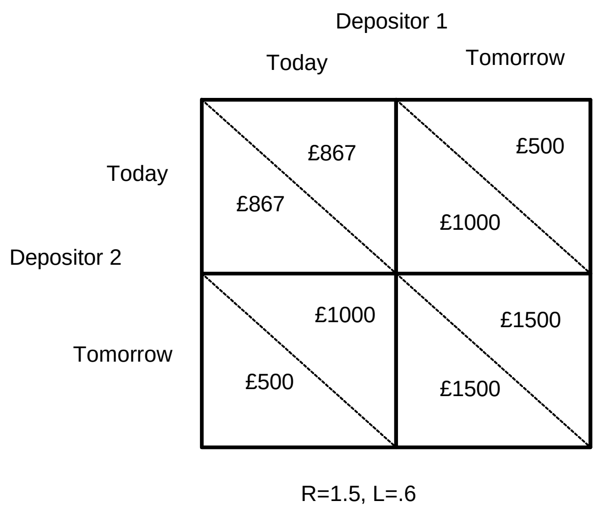

Figure 1: Bank Run game with 4 impatient and 2 patient depositors.

(ii) (5 points) If all 6 depositors withdraw today, the bank will only have 4000+2000*.6=5200 or £867 each.

If 5 depositors withdraw today (4 impatient + 1 patient), then the bank will have to liquidate 1000/.6 of its illiquid assets. This leaves (2000-(1000/.6))*1.5=500 for tomorrow. Thus, the 2x2 game can be written as that in Figure 1.

(iii) (5 points) There are two pure-strategy N.E.: both withdraw today and both withdraw tomorrow. In both cases, neither has incentive to change their strategy.

(iv) (5 points) If both patient depositors wait until tomorrow, then there will be 1000 (from+3000 to split between the 3 depositors waiting until tomorrow, yielding 4000/3. If one or two of the patient depositors withdraw today, there will be enough money to pay them the 1000 today. If only one withdraws today, then there will still be 1500 left to pay both the patient and the sick impatient depositor tomorrow since the bank would not have to liquidate any of its long-term investment. This yields the game in Figure 2.

This game now has only one Nash Equilibrium: to wait until tomorrow. There is no incentive to deviate since withdrawing today would yield only £1000. Both withdrawing today can't be a N.E. since waiting until tomorrow would increase payoff from £1000 to £1500.

3 Signalling.

Professor Samuel Levy is the lecturer in a module called "Weirdonomics". He wants to find out if students are genuinely interested in his module or simply pretending to be. (Assume that any test he writes cannot distinguish between

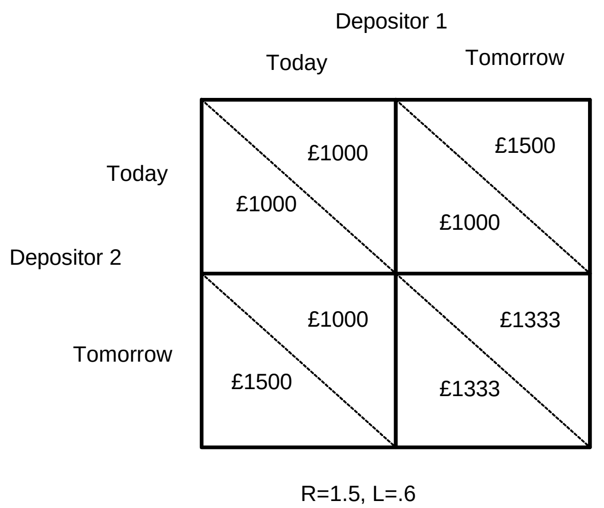

Figure 2: Bank Run Game with 3 healthy impatient depositors, 2 patient depositors and 1 sick impatient depositor.

the types.) If he knows who is who, then he would give all the students who are interested a 90 and anyone who isn't interested an 80. For giving a student a 90 who is interested or a student an 80 who is disinterested he gets 100 utils. For giving a student a 90 who is disinterested or a student a 80 who is interested he gets 0 utils.

All students prefer a 90 to a 80 by 100 utils. Interested students enjoy Samuel's lecture and have no problem show up and staying awake; in fact, they get 10 utils for it. For a disinterested student to show up to Samuel's lecture and stay awake, it will cost him or her 150 utils. Assume 2/3 of the students are genuinely interested.

(i) Is it an equilibrium for all students to show up whether they are interested or not? Is it an equilibrium for only interested students to show up and any disinterested students to stay at home?

(ii) Now assume that for a disinterested student to show up to Samuel's lecture and stay awake, it will cost him or her 50 utils. Is it an equilibrium for all students to show up whether they are interested or not? Is it an equilibrium for only interested students to show up and any disinterested students to stay at home?

3.1 Answers (out of 20 points)

(i) (10 points) The game reduces to that in Figure 3.

Notice that both types of students showing up can't be an equilibrium, since the not interested students will stay home. In fact, no matter what poor

Payoffs: Student, Professor

	Give 90	Give 80
Show up (Interested)	110, 100	10, 0
Stay home (Interested)	100, 100	0, 0
Show up (Bored)	-50, 0	-150, 100
Stay home (Bored)	100, 0	0, 100

Figure 3: Signalling game with cost of disinterested (bored) students showing up at -150 utils.

Professor Levy does, these students won't show up. If Professor Levy believes that all showing up are interested students, he will give them a 90. If this is indeed the case, the interested students will choose to go to class. Thus, there is a separating equilibrium with Levy giving those showing up a 90 and those staying home an 80.

(ii) (10 points) See the new game in Figure 4. Now there can't be a separating equilibrium since the disinterested students could show up and pretend to be interested and receive a 90. This would give them 50 utils as opposed to 0 utils. Also there can now be a pooling equilibrium where both students show up and receive a 90 (and a student that decides to not show up will receive an 80). Since 2/3 of the students are interested, Levy will have a desire to give those showing up a 90 since his expected payoff would be (2/3)*100+(1/3)*0 which is greater than giving them all an 80 with payoff (2/3)*0+(1/3)*100.

4 Price Competition.

There is a demand for Nike shoes of $15 - 2p$, where p is the price of a pair of shoes. It costs Nike 4 pounds a pair of shoes it buys.

(i) What is the profit maximizing price and quantity?

(ii) Nike decides to break itself into two companies:Nike Left and Nike Right. Yup, you guessed it. Nike Right sells only right shows and Nike Left sells only left shoes. These two companies decide prices independently and simultaneously. What is the equilibrium prices and quantity?

(iii) Explain how the monopoly price can be supported by a repeated game with discount factor β (close enough to 1). Hint: you can look at only three possibilities: $p_l = p_r = p_m/2$ (where p_l is the price of the left shoe, p_r is the price of the right shoe and p_m is the monopoly price), the profit maximizing price given the other firm chooses a price of $p_m/2$, and when one firm sets $p = 7.5$.

Payoffs: Student, Professor

	Give 90	Give 80
Show up (Interested)	**110, 100**	**10, 0**
Stay home (Interested)	**100, 100**	**0, 0**
Show up (Bored)	**50, 0**	**-50, 100**
Stay home (Bored)	**100, 0**	**0, 100**

Figure 4: Signalling game with cost of disinterested (bored) students showing up at -50 utils.

4.1 Answer (20 points):

"I like Nike but wait a minite
The neighborhood supports so put some
Money in it
Corporations owe
Dey gotta give up the dough
To da town
or else
We gotta shut 'em down (Chuck D)"

(i) (8 points) Nike wants to choose a price to maximize

$$\max_p (15 - 2p)(p - 4)$$

The first order condition is $15 - 2p_m - 2(p_m - 4) = 0$ or $p_m = 23/4 = 5.75$. Substituting into demand yields $q_m = 7/2 = 3.5$. Note profit is $\pi_m = 49/8 = 6.125$.

(ii) (8 points) If Nike breaks into a left shoe and right shoe company each with a marginal cost of 2 per shoe.

Each firm will maximize its profits separately.

$$\begin{aligned}\pi_l &= \max_{p_l}(15 - 2p_l - 2p_r)(p_l - 2) \\ \pi_r &= \max_{p_r}(15 - 2p_l - 2p_r)(p_r - 2)\end{aligned}$$

The two first-order conditions are

$$\begin{aligned}(15 - 2p_l - 2p_r) - 2(p_l - 2) &= 0 \\ (15 - 2p_l - 2p_r) - 2(p_r - 2) &= 0\end{aligned}$$

Solving yields $p_l = p_r = \frac{19}{6} = 3.167$. The quantity is then $\frac{7}{3} = 2.33$ and the per firm profit is $\frac{49}{18} = 2.72$. Notice the price goes up and the profits go down. (Everyone is worse off.)

(iii) (4 points) Call $\pi_l(p_l, p_r)$ the profit of the left shoe firm at prices p_l, p_r.

Note that $\pi_l(p_m/2, p_m/2) = \pi_m/2$. Also note that if the other firm chooses a price of 7.5, you should choose a price of 0. Hence, for a monopoly price to be supported we must have

$$\frac{\pi_m}{2(1-\beta)} \geq \max_p \pi_l(p, p_m/2) = \frac{441}{128} = 3.45$$

Or

$$\beta \geq 1 - \frac{\pi_m}{2}\frac{128}{441} = 1 - \frac{49}{16}\frac{128}{441} = \frac{1}{9}$$

So $\beta \geq 1/9$.

5 Asymmetric Information.

You are thinking buying a falafel stand from the owners. The current owners have a much better idea of the value of the firm to them than you do. (For instance, the tax forms may underrepresent their true earnings.) There is an equal chance of the firm having a value between \$75,000 to \$125,000 (and the owners know the exact value). You with your amazing University of Haifa economics education will be able to expand the business and increase profit by a factor of 5/4. What price should you offer?

5.1 Answer (20 points):

Assuming you want to maximize your expected profit than you want to maximize the probability of buying the stand times the expected profit of owning the stand given that the owners accepted your offer.

Since the current owners will accept any offer over their value, the probability them accepting an offer of p is

$$\frac{p - 75,000}{125,000 - 75,000}.$$

The expected value to the owners if accepted is the average of all values between $75,000$ and p or

$$\frac{p + 75,000}{2}.$$

Thus, the net gain is

$$\frac{5}{4}\frac{p + 75,000}{2} - p.$$

We would want to solve

$$\max_p \frac{p - 75,000}{125,000 - 75,000} \cdot \left[\frac{5}{4}\frac{p + 75,000}{2} - p\right]$$

This simplifies to

$$\max_p \frac{p - 75,000}{50,000} \cdot \left[\frac{125,000 - p}{8/3}\right]$$

Hence, $p = 100,000$ maximizes one's expected profit.

6 Subgame Perfection.

Modify the ultimatum game. Player A makes the initial proposal and player B decides to accept or reject it. If B accepts the proposal, then it stands as is. However, if there is a rejection by B, then the pie shrinks from \$10 to \$8 and B makes a proposal to A. Now, if A accepts the proposal, it stands. If A rejects it, each player get \$2. What is the subgame-perfect equilibrium of this game (assuming preferences are selfish)?

6.1 Answer (20 points):

Start with the last stage. If B knows that A will get \$2 from a rejection, B can offer A only \$2 and keeping \$6 for himself. A knowing this will off B \$6 in the first stage keeping \$4 for himself. Thus, the equilibrium is an offer of (4,6) in the first stage with an acceptance by B.

Bonus.

Guess a number between 0 and 100. The closest person to the (2/3 times the average number) wins a bonus of 1% on the final. Ties will be broken randomly.

6.2 Answer:

Average was 28.6. Two-thirds of this is 19.1. Winning guess was 21.