BS2551 Money Banking and Finance

We assume that investors (who are risk averse) measure the expected utility among risky assets by looking at the mean return and the variance of returns provided by a combination of these assets.

Since we focus on mean and variance only, we are implicitly assuming that asset returns follow a normal distribution which can be described by the mean and variance.

The normal distribution is symmetric, so probability of positive and negative returns are equally likely.

In reality the normal distribution assumption is violated, e.g with US stock returns monthly data (1871-2002): Skewness < 0, kurtosis > 3 suggesting fat tails.

Non normality in stock returns is not uncommon, especially with high frequency data (monthly, daily, tick data), leading to problems with hypothesis testing (solution: simulated critical values see Arghyrou and Gregoriou, 2006).

Mean and Variance Portfolio of an Individual Security

Probability	Security X	Security Y
0.2	0.11	-0.03
0.2	0.09	0.15
0.2	0.25	0.02
0.2	0.07	0.2
0.2	-0.02	0.06

$$E\left[X\right]=\sum_{i=1}^{N} p_i X_i = 0.2(0.11)+0.2(0.09)+0.2(0.25)+0.2(0.07) + 0.2(-0.02) = 0.1 \approx 10\%$$

$$Var\left[X\right]=\sum_{i=1}^{N}p_i\left(X_i-E\left(X\right)\right)^2=0.2(0.11-0.1)^2+0.2\left(0.09-0.1\right)$$
$$+0.2(0.25-0.1)^2+0.2(0.07-0.1)^2+0.2(-0.02-0.1)^2$$
$$=0.0076\approx0.76\%$$

Using the same methodology we compute the mean return of y = 0.08 or 8% and the variance of y as 0.00708 or 0.708%.

Asset X offers greater return but with a higher level of risk.

The notion that stock market investors have received higher mean rates of return, as compared to investors who invest in risk free assets such as Treasury Bills, for bearing higher risk is well established in financial theory.

For instance over the time period 1926-1997, returns on large US stocks had a mean of 13% with standard deviation 20.3%, while returns on TBs were significantly lower (mean=3.8%) and less volatile (st dev=3.2%).

The degree of risk exposure therefore determines expected returns not only across different types of assets but also within the asset itself.

In the case of financial markets, Malkiel (2003) points out "Risk and risk alone, decides the valuation of any stock relative to the market".

Portfolio Returns

In order to compute portfolio mean and variance we need to decide the proportion of investment on X (a%) and Y (1-a%).

If we invest half of wealth in X and half in Y: a=1, 1-a=0.5:

Return on the portfolio = 0.5(0.1) + 0.5(0.08)

=0.09 = 9%.

Variance of the portfolio = $(0.5)^2(0.0076)+(0.5)^2(0.00708)+2(0.5)(0.5)(0.0024)=0.025=0.25\%$.

Note: (0.0024) is the covariance between asset X and asset Y.

The benefits of portfolio diversification are now apparent. By investing in both risky assets X and Y risk is reduced. The variance of the portfolio is 0.25%, whereas the variance of asset X is 0.76% and the variance of asset Y is 0.708%.