### Differentiation

<table>
<thead>
<tr>
<th>Function</th>
<th>Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = f(x)$</td>
<td>$\frac{dy}{dx} = f'(x)$</td>
</tr>
<tr>
<td>$k$, constant</td>
<td>0</td>
</tr>
<tr>
<td>$x$</td>
<td>1</td>
</tr>
<tr>
<td>$x^2$</td>
<td>$2x$</td>
</tr>
<tr>
<td>$x^3$</td>
<td>$3x^2$</td>
</tr>
<tr>
<td>$x^n$, any constant $n$</td>
<td>$nx^{n-1}$</td>
</tr>
<tr>
<td>$e^x$</td>
<td>$e^x$</td>
</tr>
<tr>
<td>$e^{kx}$</td>
<td>$ke^{kx}$</td>
</tr>
<tr>
<td>$e^{f(x)}$</td>
<td>$f'(x)e^{f(x)}$</td>
</tr>
<tr>
<td>$\ln x$</td>
<td>$1/x$</td>
</tr>
<tr>
<td>$\ln(kx)$</td>
<td>$1/kx$</td>
</tr>
<tr>
<td>$\ln(f(x))$</td>
<td>$f'(x)/f(x)$</td>
</tr>
</tbody>
</table>

- **The sum–difference rule:** $\frac{d}{dx}(u(x) \pm v(x)) = \frac{du}{dx} \pm \frac{dv}{dx}$
- **Constant multiples:** $\frac{d}{dx}(k \cdot f(x)) = k \cdot \frac{df}{dx}$ for $k$ constant

- **The product rule:** $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
- **The quotient rule:** $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - \frac{dv}{dx}u}{v^2}$

### Integration

<table>
<thead>
<tr>
<th>Function</th>
<th>Integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$, (any) constant $c$</td>
<td>$kx + c$</td>
</tr>
<tr>
<td>$x$</td>
<td>$\frac{x^2}{2} + c$</td>
</tr>
<tr>
<td>$x^2$</td>
<td>$\frac{x^3}{3} + c$</td>
</tr>
<tr>
<td>$x^n$, $n \neq -1$</td>
<td>$\frac{x^{n+1}}{n+1} + c$</td>
</tr>
<tr>
<td>$x^{-1}$, $1/x$</td>
<td>$\ln</td>
</tr>
<tr>
<td>$e^x$</td>
<td>$e^x + c$</td>
</tr>
<tr>
<td>$e^{kx}$</td>
<td>$\frac{e^{kx}}{k} + c$</td>
</tr>
</tbody>
</table>

### Graphs of Common Functions

- **Linear:** $y = mx + c$; $m$ = gradient; $c$ = vertical intercept

  - Positive gradient: $y = mx + c$
  - Negative gradient: $y = mx + c$

- **Exponential functions**
  - $e = 2.7183$ is the exponential constant
  - Graph of $y = e^x$ showing exponential growth
  - Graph of $y = e^{-x}$ showing exponential decay

- **Quadratic functions**
  - $y = ax^2 + bx + c$

  - If $a$ is positive
  - If $a$ is negative

- **Total cost functions**
  - $TC = a + bq - cq^2 + dq^3$

- **Inverse functions**
  - $y = a/x = ax^{-1}$

  - Example: Unit price elasticity of demand $q = a/p = ap^2$
### Arithmetic

When multiplying or dividing positive and negative numbers, the sign of the result is given by:

- + and + gives +  e.g. 6 x 3 = 18; 21 ÷ 7 = 3
- – and + gives –  e.g. (–6) x 3 = –18  (–21) ÷ 7 = –3
- + and – gives –  e.g. 6 x (–3) = –18  21 ÷ (–7) = –3
- – and – gives +  e.g. (–6) x (–3) = 18  (–21) ÷ (–7) = 3

#### Order of calculation

First: brackets  
Second: x and ÷  
Third: + and –

#### Fractions

- Fraction = \( \frac{\text{numerator}}{\text{denominator}} \)

#### Adding and subtracting fractions

To add or subtract two fractions, first rewrite each fraction so that they have the same denominator. Then, the numerators are added or subtracted as appropriate and the result is divided by the common denominator: e.g.

\[
\frac{4}{5} + \frac{3}{10} = \frac{16 + 15}{20} = \frac{31}{20}
\]

#### Multiplying fractions

To multiply two fractions, multiply their numerators and then multiply their denominators: e.g.

\[
\frac{3}{7} \times \frac{5}{11} = \frac{15}{77}
\]

#### Dividing fractions

To divide two fractions, invert the second and then multiply: e.g.

\[
\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}
\]

### Algebra

#### Removing brackets

\[
a(b + c) = ab + ac \quad a(b - c) = ab - ac
\]

\[
(a + b)(c + d) = ac + ad + bc + bd
\]

\[
(a + b)^2 = a^2 + b^2 + 2ab; \quad (a - b)^2 = a^2 + b^2 - 2ab
\]

#### Formula for solving a quadratic equation

If \( ax^2 + bx + c = 0 \), then

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

#### Laws of indices

\[
a^m \times a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \quad (a^m)^n = a^{mn}
\]

\[
a^0 = 1 \quad a^{-m} = \frac{1}{a^m} \quad a^{m/n} = \sqrt[n]{a^m}
\]

#### Laws of logarithms

\[
y = \log_a x \quad \text{means} \quad b^y = x \quad \text{and} \quad b \text{ is called the base}
\]

e.g. \( \log_{10} 2 = 0.3010 \) means \( 10^{0.3010} = 2 \) to 4 sig figures

Logarithms to base e. denoted \( \log_e \) or alternatively \( \ln \), are called natural logarithms. The letter \( e \) stands for the exponential constant, which is approximately 2.7183.

\[
\ln AB = \ln A + \ln B; \quad \ln \frac{A}{B} = \ln A - \ln B; \quad \ln A^n = n \ln A
\]

### Proportion and Percentage

To convert a fraction into a percentage, multiply by 100 and express the result as a percentage. An example is:

\[
\frac{5}{8} \text{ as a percentage is } \frac{5}{8} \times 100 = 62.5\%
\]

#### Some common conversions are

\[
10\% = \frac{1}{10} \quad 25\% = \frac{1}{4} \quad 50\% = \frac{1}{2} \quad 75\% = \frac{3}{4}
\]

#### Ratios

Ratios are simply an alternative way of expressing fractions. Consider dividing £200 between two people in the ratio of 3:2. This means that for every £3 the first person gets, the second person gets £2. So the first gets \( \frac{3}{5} \) of the total (i.e. £120) and the second gets \( \frac{2}{5} \) (i.e. £80).

Generally, to split a quantity in the ratio \( m:n \), the quantity is divided into \( m/(m+n) \) and \( n/(m+n) \) of the total.

### Sigma Notation

The Greek capital letter sigma \( \Sigma \) is used as an abbreviation for a sum. Suppose we have \( n \) values: \( x_1, x_2, \ldots, x_n \) and we wish to add them together. The sum

\[
x_1 + x_2 + \ldots + x_n \quad \text{is written} \quad \sum_{i=1}^{n} x_i
\]

Note that \( i \) runs through all integers (whole numbers) from 1 to \( n \). So, for instance

\[
\frac{1}{3} \sum_{i=1}^{3} x_i \quad \text{means} \quad x_1 + x_2 + x_3
\]

#### Example

\[
\sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2
\]

This notation is often used in statistical calculations. The mean of the \( n \) quantities, \( x_1, x_2, \ldots, x_n \) is

\[
\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}
\]

#### The variance

\[
\text{var}(x) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} x_i^2 - \bar{x}^2}{n}
\]

i.e. the mean of the squares minus the square of the mean

The standard deviation (sd) is the square root of the variance:

\[
sd(x) = \sqrt{\text{var}(x)}
\]

Note that the standard deviation is measured in the same units as \( x \).

### The Greek Alphabet

<table>
<thead>
<tr>
<th>Greek</th>
<th>English</th>
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</thead>
<tbody>
<tr>
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<td>iota</td>
<td>( \rho )</td>
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